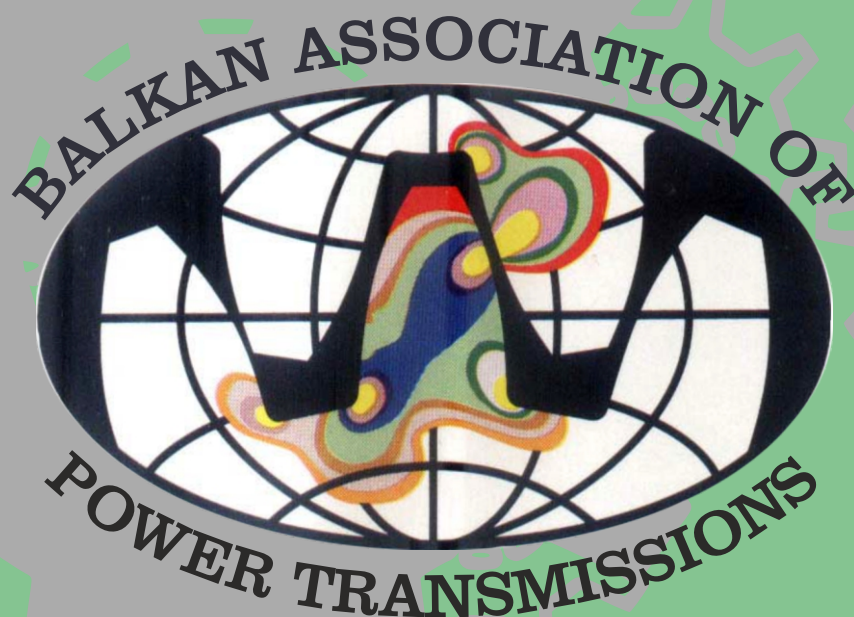


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04-07.09.2023, VARNA, BULGARIA



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YEAR I

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Theoretical analysis of transmission error in rack and pinion systems

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Abstract. Rack and pinion drive systems are widely used in machine tools with long travel distances because the stiffness is independent of the travelled distance, in contrast to other drive systems such as ballscrews. Although the inherent backlash problem of gear systems has been solved by means of utilizing two pinions independently preloaded, the time-varying mesh stiffness causes periodic position differences between the motor encoder and table position, which is known as transmission error, and may lead to vibrations and dynamic load. Few experimental works have analyzed such transmission error, but there are no theoretical approaches in the scientific literature. Therefore, this work aims to first extend previous analytical and approximate equations for mesh stiffness of gearing to rack and pinion systems and validate them with finite element methods. Finally, the effect of geometry variations, such as pressure angle and tooth thickness tolerance, on transmission error is discussed.

1 Introduction

Rack and pinion drive systems are widely employed in machine tools with long travel distances due to their inherent advantages, including high stiffness and independence from the travelled distance. Unlike other drive systems, such as ballscrews, where stiffness can vary based on the distance travelled, rack and pinion systems provide consistent stiffness throughout the entire travel range [1]. This characteristic makes them particularly suitable for applications requiring precise and rigid positioning.

However, despite the advantages of rack and pinion systems, they are not immune to certain challenges. One significant issue is the presence of backlash inherent to any geared system. To mitigate such source errors, one notable approach involves the utilization of two pinions with independent preload, which effectively addresses the inherent backlash problem in gear systems [2, 3]. By independently preloading the pinions, the inherent backlash can be minimized, ensuring a more precise and reliable operation of the rack and pinion drive system [4].

Another significant issue is the transmission error, which refers to periodic position differences between the motor encoder and the table position during operation. Transmission error occurs due to the time-varying mesh stiffness inherent in gear systems [5-9]. These periodic position differences, if not properly controlled, can lead to detrimental effects such as dynamic load, vibrations, and reduced positioning accuracy, impacting the overall performance of the system.

While some experimental works have explored the analysis of transmission error in rack and pinion systems

[10], there is a significant gap in the existing literature when it comes to theoretical approaches. The few theoretical approaches in rack and pinion systems are related to other applications, such as pure rolling [11], automotive engine [12], and steering rack [13] or ship lift mechanism [14], but they are not focused on transmission error. The scientific literature lacks comprehensive theoretical investigations and models specifically addressing transmission error in rack and pinion systems. Therefore, there is a need to develop analytical and approximate equations to accurately quantify the mesh stiffness of gearing in rack and pinion systems.

The objective of this research is to bridge this gap and provide a theoretical analysis of transmission error in rack and pinion systems. The primary aim is to extend previous analytical and approximate equations for mesh stiffness of gearing to specifically address rack and pinion systems. These equations will be validated through finite element methods, ensuring the accuracy and reliability of the theoretical model. Furthermore, the research will explore the effect of geometry variations, such as pressure angle and tooth thickness tolerance, on transmission error, shedding light on potential strategies for mitigating this issue.

By conducting a comprehensive theoretical analysis, this research contributes to the understanding and optimization of rack and pinion drive systems, leading to enhanced performance, reduced vibrations, and improved positioning accuracy. The findings of this study will provide valuable insights for engineers and researchers involved in the design, development, and control of machine tools and other applications utilizing rack and pinion systems.

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2 Analytical model for the mesh stiffness

The mesh stiffness of a tooth pair, also called single mesh stiffness (SMS) and denoted by K_M , can be expressed as [5]:

$$K_M = \frac{1}{k_{x1}} + \frac{1}{k_{s1}} + \frac{1}{k_{n1}} + \frac{1}{k_{R1}} + \frac{1}{k_{x2}} + \frac{1}{k_{s2}} + \frac{1}{k_{n2}} + \frac{1}{k_{R2}} + \frac{1}{k_H} \quad (1)$$

where k_x is the bending stiffness, k_s the shear stiffness, k_n the compressive stiffness, k_R the gear body stiffness, k_H the contact stiffness, and subscripts 1 and 2 denote the pinion and the rack, respectively.

The bending, shear, and compressive stiffnesses can be assessed from the potential energy method, which is a widely accepted analytical approach [5]. This method treats the gear tooth as a variable-section cantilever beam fixed at its dedendum diameter and estimates its total potential energy by solving analytical expressions derived from mechanics of materials. Accordingly, the equations for three components are:

$$\frac{1}{k_x} = \frac{12}{Eb} \int_{y_p}^{y_c} (y_c - y) \cos \alpha_c - r_c \sin \frac{\gamma_c}{2} \sin \alpha_c \left[\frac{dy}{e^3(y)} \right] \quad (2)$$

$$\frac{1}{k_n} = \frac{1}{Eb} \int_{y_p}^{y_c} \sin^2 \alpha_c \frac{dy}{e(y)} \quad (3)$$

$$\frac{1}{k_s} = C_s \frac{1}{Gb} \int_{y_p}^{y_c} \cos^2 \alpha_c \frac{dy}{e(y)} \quad (4)$$

where E is the modulus of elasticity of steel, G is the transverse modulus of elasticity, b is the face width, and C_s is the shear stress correction factor, which for a rectangular section takes the value $C_s = 1.2$. Subscript c denotes the contact point and therefore r_c is the radius of the contact point, γ_c the angular thickness at the corresponding tooth section, and α_c the load angle at this contact point. Finally, y is a linear coordinate along the tooth centerline from the center of the gear (gear rotation axis), y_p corresponds to the embedded tooth section at the dedendum circle, y_c corresponds to the load tooth section, and $e(y)$ is the chordal thickness at tooth section described by y . For the tooth rack, the origin of coordinate y is an arbitrary plane parallel to the pitch plane, the angular thicknesses are equal to the corresponding chordal thicknesses, and the load angle is equal to the standard normal pressure angle α_n at any contact point.

The contact stiffness can be assessed from the Hertz approach or Weber-Banaschek approach. The Hertz approach results in [5]:

$$k_H^{-1} = \frac{4}{\pi} \frac{1 - \nu^2}{Eb} \quad (5)$$

ν being the Poisson ratio. The Weber approach is more complicated and can be found in [6]. Finally, the gear

body stiffness can be calculated according to Weber-Banaschek [6] or Sainsot [7] contributions.

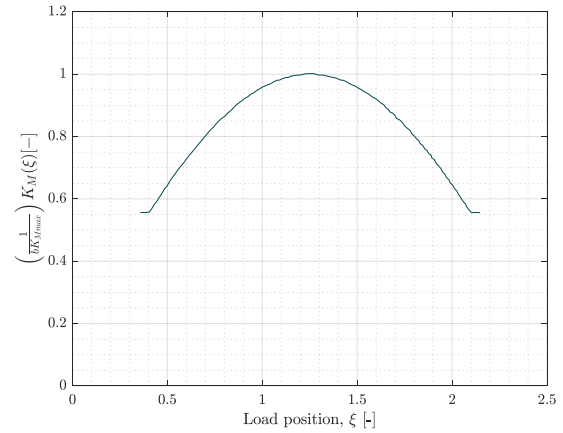


Fig. 1. Typical shape of single mesh stiffness curve.

Different combinations of those approaches were studied in [8], but in all the cases the curve of single mesh stiffness presents a shape as the one in Figure 1.

This curve can be accurately approximated by the equation [5]:

$$K_M(\xi) = K_{Mmax} \cos(b_0(\xi - \xi_m)) \quad (6)$$

where K_{Mmax} is the SMS at the midpoint of the path of contact, subscript m denotes this midpoint, and:

$$b_0 = \frac{1}{2} \left(\kappa_1 + \frac{\varepsilon_\alpha}{2}^2 - \kappa_2 \right)^{-1/2} \quad (7)$$

$$\xi = \frac{z_1}{2\pi} \frac{r_{c1}^2}{r_{b1}^2} - 1 \quad (8)$$

in which ε_α is the contact ratio, z_1 the number of teeth on pinion, and r_b the base radius. Coefficients κ_1 and κ_2 are introduced to adjust the SMS curve to eq. (6). Values for κ_1 and κ_2 for different combinations of contact stiffness and gear body stiffness can be found in [8]. In all the cases, the value of K_{Mmax} should be obtained from numerical calculations or FE analyses. Obviously, K_{Mmax} depends on the geometry of the meshing gears and the specific considered combination of approaches, but not on the applied torque.

It is observed that the rotation of the pinion $\Delta\theta$ is related with the parameter ξ as follows:

$$\Delta\theta = \frac{2\pi}{z_1} \Delta\xi \quad (9)$$

3 Finite element model

A quasi-static two-dimensional finite element model (FEM) is developed to analyze and assess the impact of transmission error in rack and pinion systems. The numerical models are automatically generated using a specialized gear mesh generator [15], and the FEM models are computed using *Abaqus* solver.

The generation of rack and pinion mesh follows the well-established methodology introduced by Litvin [16].

This methodology entails assigning a higher density of elements to the teeth compared to the body of the pinion, as depicted in Figure 2. Additionally, a progressive mesh density, known as bias factor, is determined from the symmetry axis of the tooth to the contact area [17]. To accurately capture the behavior of the system, four-node, isoparametric, arbitrary quadrilateral elements suitable for plane stress applications are employed (specifically, element type CPS4 in the *Abaqus* software [18]).

Regarding the boundary conditions, a prescribed translation velocity is imposed on the rack (v) to simulate fixed linear motion, while a torque (T) is applied to the pinion. To mitigate the influence of boundary conditions, all the teeth of the pinion are taken into consideration in the simulations [19]. Moreover, in order to avoid preloading of the teeth during contact initiation, a preliminary interference was introduced between meshing teeth. It induces a small angular deviation ϑ_0 , which should be accounted for comparisons between FEM and analytical models.

The transmission error $TE(\vartheta)$ is determined by calculating the difference between the computed rotation of the pinion ϑ_p^{FEM} and the theoretical rotation resulting from the prescribed translation of the rack ϑ_p^T , as described in eq. (10). This calculation allows for the quantification of the deviation or variation between the actual motion of the pinion and the expected motion based on the translation of the rack, accounting for the deformations arising from mesh stiffness.

$$TE(\vartheta) = \vartheta_p^{FEM} - \vartheta_p^T \quad (10)$$

where:

$$\vartheta_p^T = \frac{v \cdot t}{\pi \cdot m_n \cdot z_1} \quad (11)$$

Here, t represents the simulated time (with a time increment employed of 1 ms in this specific case), and m_n denotes the normal module.

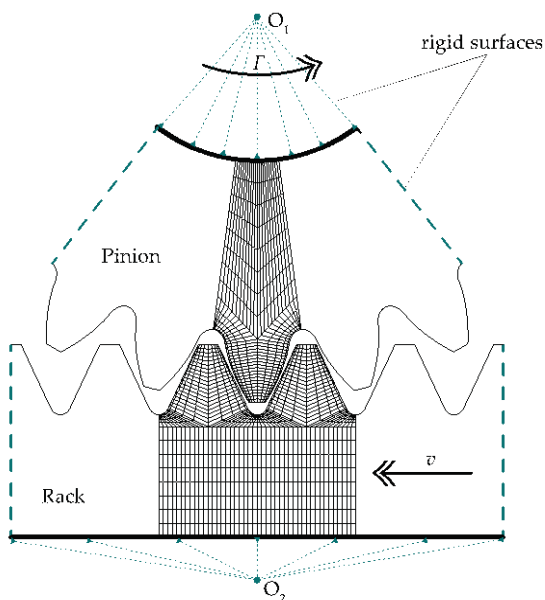


Fig. 2. Finite element mesh detail and boundary conditions of the numerical model.

4 Approximate equations for the time-varying mesh stiffness and transmission error

From the hypothesis of minimum elastic potential energy, the theoretical load at a specific tooth pair F_i at the load position described by ξ is [5]:

$$F_i(\xi) = \frac{K_{Mi}(\xi)}{\sum_j K_{Mj}(\xi)} F_T = \frac{K_{Mi}(\xi)}{\sum_j K_M(\xi + j)} F_T \quad (12)$$

where F_T is the total transmitted load and the sum is extended to all the tooth-pairs in simultaneous contact.

Accordingly, the theoretical tooth-pair deflection δ_i at any contact position will be given by [5]:

$$\delta_i(\xi) = \frac{F_i(\xi)}{K_{Mi}(\xi)} = \frac{F_T}{\sum_j K_M(\xi + j)} \quad (13)$$

As the right term does not depend on i , the tooth pair deflection is equal for all the tooth-pairs in contact and describes the delay of the driven gear with respect to the driving gear, as a distance measured on the pressure line. Obviously, the relation between the delay distance and the angle transmission error is:

$$TE(\xi) = \frac{\delta(\xi)}{r_{b1}} = \frac{F_T}{r_{b1} \sum_j K_M(\xi + j)} \quad (14)$$

$$TE(\vartheta) = \frac{F_T}{r_{b1} \sum_j K_M(\frac{z_1}{2\pi} \xi - \vartheta_0 + j)} \quad (15)$$

Equations (12) to (15) correspond to the theoretical contact model and are valid for weakly loaded teeth. For heavily loaded teeth, the delay of the driven gear results in an earlier start of contact and a delayed end of contact [20]. These additional contact intervals depend on the transmitted load and result in an affective contact ratio greater than the theoretical one. As described in [20], the distance δ_G that the driving tooth should approach to (or move away) the driven one to start (end) the contact at a given meshing position ξ , can be expressed as:

$$\delta_G(\xi) = \left(\frac{2\pi}{Z_1}\right)^2 C_{p-inn} r_{b1} (\xi_{inn} - \xi)^2 \quad (16)$$

$$\delta_G(\xi) = \left(\frac{2\pi}{Z_1}\right)^2 C_{p-out} r_{b1} (\xi - \xi_{out})^2 \quad (17)$$

where ξ_{inn} and ξ_{out} are the inner and outer limits of the theoretical contact interval, and C_{p-inn} and C_{p-out} can be computed as described in [20]. Accordingly, considering that $\delta_G(\xi) = 0$ inside the theoretical contact interval, the load transmitted by the tooth pair is:

$$F(\xi) = K_M(\xi) (\delta(\xi) - \delta_G(\xi)) \quad (18)$$

the total transmitted load will be:

$$\begin{aligned} F_T &= \sum_j K_M(\xi + j) (\delta(\xi) - \delta_G(\xi + j)) \\ &= \delta(\xi) \sum_j K_M(\xi + j) - \sum_j K_M(\xi + j) \delta_G(\xi + j) \end{aligned} \quad (19)$$

and the tooth-pair deflection:

$$\delta(\xi) = \frac{F_T + \sum_j K_M(\xi + j)\delta_G(\xi + j)}{\sum_j K_M(\xi + j)} \quad (20)$$

from which the transmission error will be given by:

$$TE(\xi) = \frac{F_T + \sum_j K_M(\xi + j)\delta_G(\xi + j)}{r_{b1} \sum_j K_M(\xi + j)} \quad (21)$$

Finally, the time varying meshing stiffness can be expressed as:

$$K_T(\xi) = \frac{F_T}{\delta(\xi)} \quad (22)$$

Equations (18), (21), and (22) describe the evolution of the load sharing, transmission error, and meshing stiffness along the path of contact under actual load conditions. The limits of the actual contact interval (extended contact interval), $\xi_{\min}$ and $\xi_{\max}$, can be obtained from eq. (18), by doing $F(\xi) = 0$. The actual contact ratio will be given by:

$$\varepsilon_{\alpha-ext} = \xi_{\max} - \xi_{\min} \quad (23)$$

5 Results and discussion

5.1. Case studies

Table 1 summarizes the geometrical characteristics of the rack and pinion system under consideration. Five different torque levels are applied, ranging from 50 Nm to 150 Nm. The material selected for the pinion and rack is a steel with 206 GPa Young modulus and 0.3 Poisson coefficient.

Three case studies have been selected with the aim of validating the proposed model for rack and pinion systems. The theoretical contact ratio of each case is respectively, 1.657, 1.791 and 1.657. First, Case 2 focuses on studying the effect of pressure angle on the transmission error, aiming to observe the effect of the contact ratio. Subsequently, Case 3 introduces the influence of tooth thickness tolerance, serving to demonstrate the capability of the proposed model in accommodating tooth geometry modifications.

5.2. Model validation

To apply the extended contact model to the contact in rack-pinion systems, it is necessary to adjust the coefficients κ_1 and κ_2 of the single mesh stiffness curve (eqs. (6) and (7)). As the flexibility of the gear body affects the SMS, the coefficients κ_1 and κ_2 will be influenced by the pinion rim thickness and pinion size. It has been proved that κ_1 and κ_2 for internal and external

gears are different, even for the same approaches of the contact and gear body stiffnesses [21]. In addition, analytical approaches of Weber [6] and Sainsot [7] are not valid for very high number of teeth, as frequently occurs for internal gears, and even more for infinite teeth, as the rack case.

After a correlation study, the adjusted values of the coefficients κ_1 and κ_2 for rigid body tooth pinion and rack body height around $1.5 m_n$ are the following:

$$\kappa_1 = 1.661 \quad (24)$$

$$\kappa_2 = 2.233 \quad (25)$$

These values are not far from those of rigid body pinion and wheel (approach IV in [8]) where $\kappa_1 = 1.56$ and $\kappa_2 = 2.00$, indicating a reasonable result.

Moreover, the SMS at the midpoint of the path of contact (K_{Mmax}) has been calculated through FEM analysis for the three cases, with stiffness units expressed in N/mm:

$$K_{Mmax}^1 = 5.076 \cdot 10^5 \quad (26)$$

$$K_{Mmax}^2 = 4.814 \cdot 10^5 \quad (27)$$

$$K_{Mmax}^3 = 4.784 \cdot 10^5 \quad (28)$$

These values are consistent with other findings in the literature for similar gear geometries [22] and capture the distinctive variations among the case studies. Specifically, the first case exhibits the highest stiffness due to a higher pressure angle, whereas the last case displays the lowest stiffness due to a smaller tooth root thickness.

Upon calculating the specific coefficients for the proposed analytical model (eq. 24-28), a comparison between the analytical model and Finite Element Method (FEM) results is presented in Figure 3. It is important to note that the coordinate system of the FEM curves has been adjusted to align with the analytical curves, compensating for the initial offset introduced in the FEM during contact initiation.

The results reveal that the proposed approximate model accurately represents both the overall values and the shape of the transmission error in rack and pinion systems. The model effectively captures the characteristics of the single tooth contact region and the contact ratio. Furthermore, the peak-to-peak error, which contributes to the dynamic load, exhibits a strong correlation between the analytical model and the numerical FEM, with maximum error lower than 5% for all the analyzed cases, as depicted in Figure 4.

Table 1. Geometry parameters of the rack and pinion system.

	Symbol	Case 1	Case 2	Case 3
Number of teeth	z_1	24		
Normal module	m_n	2.116 mm		
Pressure angle	α_n	22.5°	20°	22.5°
Face width	b	25 mm		
Profile shift	x_1	0		
Operating center distance	a_w	25.392 mm		
Reference profile - ISO 53	[-]	A (1.25/0.25/0.38)		
Tolerance - DIN 3967	[-]	0	0	- 90 μ m

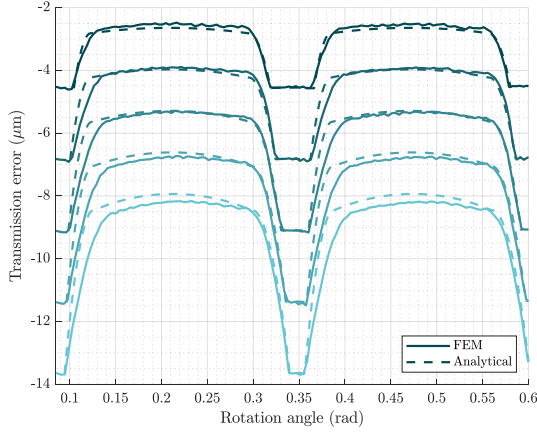


Fig. 3. Transmission error results for the FEM (continuous line) and analytical model (discontinuous line) for Case 1.

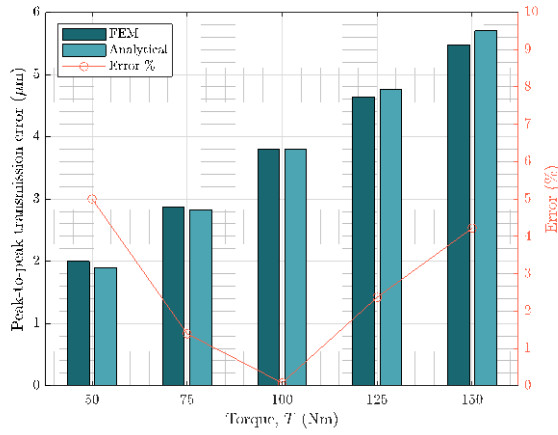


Fig. 4. Peak-to-peak transmission error for different torque and maximum error between analytical and FEM results for Case 1.

Looking into the differences, it can be observed that the analytical model predicts a symmetrical response in the transmission error, whereas the FEM results exhibit a different response during the loading and unloading phases from single to double teeth contact regions. These findings are in agreement with the results reported in [22], where an increase in such asymmetry was observed when increasing the transmission ratio. In the context of rack and pinion systems, where the transmission ratio is theoretically infinite, this represents the extreme condition and that is why it is even more noticeable. However, it is worth noting that for practical applications, these differences can be considered negligible, even under such extreme transmission ratio conditions.

5.3. Influence of the pressure angle

Following the validation of the analytical model, an investigation into the impact of the pressure angle is conducted. The pressure angle influences the amplitude of SMS and the contact ratio, which results in a slightly different curves of mesh stiffness and transmission error. The results for a pressure angle of 20° are presented in Figure 5. Notably, both models exhibit remarkably similar results, with peak-to-peak transmission error values consistently below 10% (as illustrated in Figure 6).

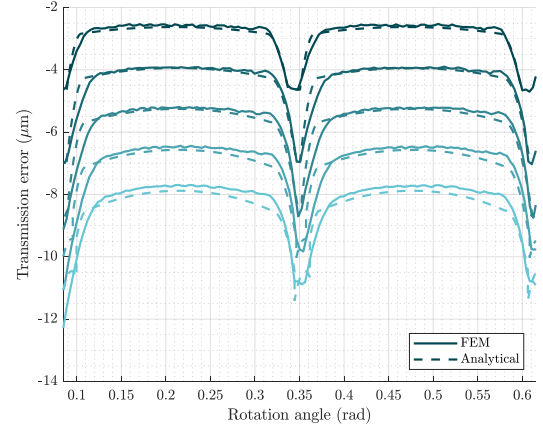


Fig. 5. Transmission error results for the FEM (continuous line) and analytical model (discontinuous line) for Case 2.

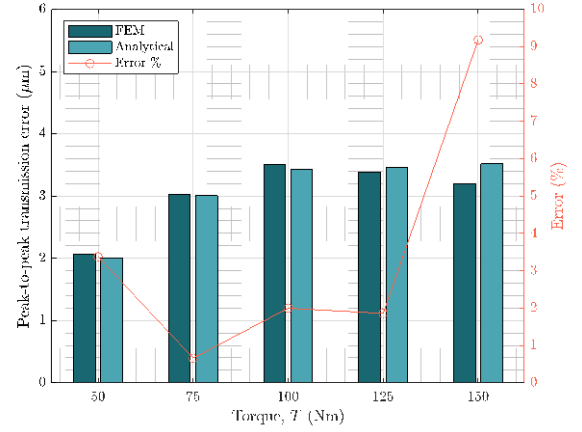


Fig. 6. Peak-to-peak transmission error for different torque and maximum error between analytical and FEM results for Case 2.

Furthermore, when compared to case study 1, it is worth noting that the initial peak-to-peak transmission error exhibits a slight increase, which aligns with the expected trend attributed to lower stiffness values. However, it is also noticeable that the contact ratio is higher in this case study, resulting in a shorter single tooth contact region. Consequently, for torques exceeding 100 Nm, three teeth come into contact, leading to the stabilization of the peak-to-peak transmission error. Both models successfully capture this phenomenon, although some discrepancies are observed within these regions. Probably, the FEM model is slightly less accurate for three tooth-pairs in contact; however, discrepancies are small, and results can be considered acceptable.

5.4. Influence of thickness tolerance

Finally, the effect of tooth thickness is considered in the analysis. Figure 7 shows the comparison between analytical and FEM results for the transmission error, while Figure 8 depicts the corresponding peak-to-peak transmission error. Notably, the obtained results align with previous findings, demonstrating a strong agreement between both models. When comparing it to Case 1, the stiffness of the tooth is lower and therefore, slight increase of the peak-to-peak transmission errors are observed.

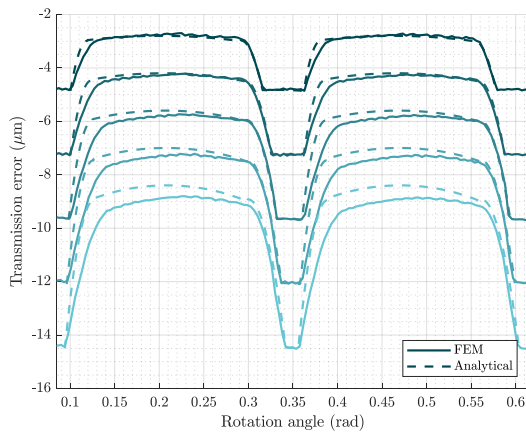


Fig. 7. Transmission error results for the FEM (continuous line) and analytical model (discontinuous line) for Case 3.

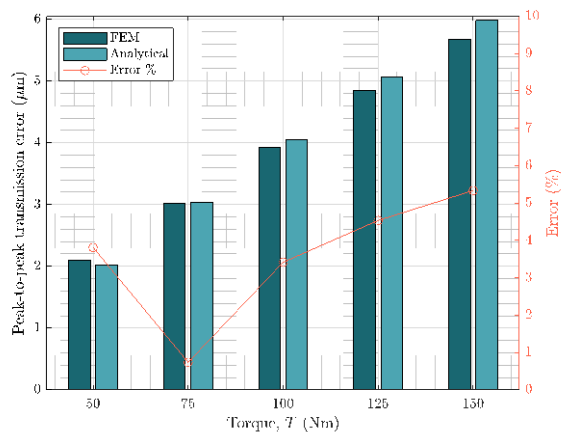


Fig. 8. Peak-to-peak transmission error for different torque and maximum error between analytical and FEM results for Case 3.

6 Conclusions

Based on the results shown in the present study, the following conclusions can be drawn:

- The proposed analytical model effectively represents the transmission error in rack and pinion systems, capturing both the overall values and the shape of the transmission error curve.
- The effect of different factors on the transmission error has been investigated. The impact of variables such as pressure angle, tooth thickness, and contact ratio has been examined. The results demonstrate that these factors influence the magnitude and characteristics of the transmission error.
- The analysis of the transmission error under different scenarios and conditions provides valuable insights for understanding the dynamic behavior and performance of rack and pinion systems. These findings contribute to improving the design and operational efficiency of these systems.

In conclusion, the results highlight the accuracy of the analytical model, the influence of various factors on the transmission error, and the significance of tooth stiffness. This knowledge enhances our understanding of rack and pinion systems and enables informed decision-

making for design improvements and performance optimization.

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Energy efficiency analysis of double suction centrifugal pumps with new impeller geometry

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Abstract. The development of energy saving determines the importance of improving the efficiency of equipment that uses most of the energy consumed. Pumping equipment is one of the most significant consumers of electrical energy. Thus, the drive of pumps (mainly centrifugal pumps) at some CHPPs consumes up to 10 % of the total energy produced at the plant [1]. In general, the total share of consumption of pumping equipment operated in industry, according to various estimates, is from 15 to 25 % of all generated electricity. In this paper, a new impeller configuration (with variable blade curvature) for double suction centrifugal pumps with higher efficiency was proposed. The head and efficiency of centrifugal pump with different vane grids were compared. The main conclusions are that the static pressure and relative velocity have increased in the modernised impeller, leading to an increase in hydraulic and overall efficiency, the kinetic energy of turbulence has less pulsation. the operating point of the pump has shifted. In spite of the fact that the average integral efficiency has increased in the modified pump, the operating point of the pump is shifted to the side of decreasing flow by 1.5%. The theory of the variable curvature vane system was verified by computational methods.

Key words: centrifugal pump, experiment, power, pressure, efficiency, computational methods

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1 Introduction

There are currently two main ways of solving the problem of improving the efficiency of pumping equipment: the design and manufacture of new pumps, more advanced than the previous generation, or the modernisation of existing equipment. The second way - modernisation of existing pumps - provides an opportunity for a relatively quick and low-cost set of measures to increase the efficiency and reliability of pumping equipment

When analysing and researching the most promising methods, we have identified directions for improving the flow part of the impeller. So in the work [2] studied the influence of impeller angles of double suction, it was found that, changing the angles of blade inclination can cause pressure fluctuations with high amplitude vibration of centrifugal pumps with double suction. The authors conducted experiments on pressure fluctuations are conducted for five configurations of impeller with different angles of inclination using the same test bench system. The results show that the offset angles have a negligible effect on the head characteristics and efficiency. The inclination angle has a minor effect on the pressure fluctuations in the suction chamber, while it has a major effect on the pressure fluctuations in the volute casing, especially in

the aspect of decreasing amplitude at the blade pass frequency.

In the direction of increasing the efficiency by modernising the vane system, the author [3] proposes a centrifugal pump impeller which contains at least two vanes with different entry angle β_{11} . All impeller blades are arranged with a constant external pitch α and have the same exit angle β_{12} . In a particular case, each impeller blade corresponds to a blade with the same entry angle β_{11} arranged symmetrically about the centre of the impeller. The impeller may include three pairs of blades with different entry angles β_{11} . The efficiency of the pump is increased in the area of flow values different from the design value.

One of the possible directions for solving the problem of increasing the efficiency and energy efficiency of CNs is the idea of using double-deck RCs, which has attracted the attention of researchers both in the field of pump engineering [4-7]. The application of wheels with double-deck grating was firstly suggested by Pfleiderer. He points out the possibility of using a wheel of this design when the outlet constriction factor is significant. The first practical attempts to implement and test this idea in order to increase the efficiency of the stage and

improve the shape of the pressure characteristic did not lead to the expected result.

The authors of the work suggest replacing the spatial vane system of the centrifugal impeller with a low ($ns = 65$) specific speed of rotation with cylindrical blades, because such working bodies most intensively carry out the process of energy transfer from the impeller to the working fluid. This will provide an increase in the impeller head and simplification of its manufacturing technology. [8]

2 Materials and methods

2.1 Basic Parameters of the Impeller

The two main components of a centrifugal pump are the impeller and the casing. The other components are the suction nozzle, discharge nozzle, shaft, bearing, wear rings, stuffing box and mechanical seal.

A double suction impeller is essentially no different from two single suction impellers, arranged closely together on a horizontal shaft and supported by supports on both sides. This design allows for increased capacity without increasing the impeller diameter. This type allows the fluid to enter the impeller eyelet on both sides. This action can be symmetrical about the centre line of the double suction impeller. The symmetry of the impeller greatly improves its hydraulic balance or no axial thrust force. The centre helix serves both impellers and leads through one diffuser to the outlet flange [9].

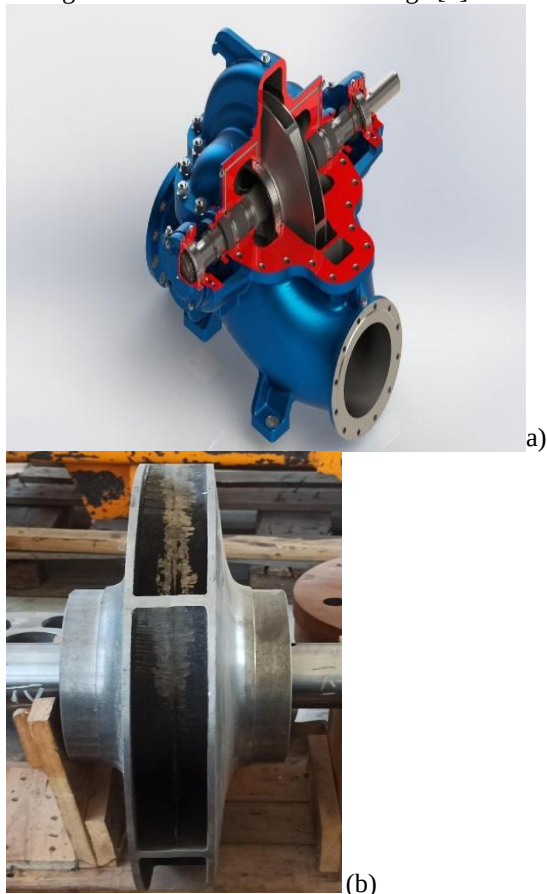


Figure 1. CNP 630-90 pump a) general view of the pump b) double inlet impeller

This arrangement often results in higher efficiency because it reduces the friction at the back of the impellers, the friction losses of the discs, and because the specific speed of each impeller is sometimes greater when the flow is split in two, favourable. As a result, double suction volute pumps can generate higher pressures than single suction centrifugal pumps [10].

The investigated pump of CNP 630-90 brand is manufactured by KARLSKRONA company (Kazakhstan). Technical characteristics of the pump and geometrical characteristics of the impeller are given in

Table 1. Technical characteristics of the pump and geometrical characteristics of the impeller

Pump volumetric flow rate nominal, QH, m ³ /h	630
Pump pressure, H _H , m	90
Shaft speed rev/min	1450
Working fluid	water
Density ρ Kg/m ³	1050
Impeller inlet diameter, D1, mm	225
Impeller outlet diameter D2, mm	525
Impeller exit width b2, mm	50
Number of blade, Z	6
Volute inlet width b1, mm	70

The method of industrial tests for determining the efficiency of the pump consists in determining the pressure and energy characteristics of the pump at five points in the working range of the pump, which is usually a flow rate from 0.5 QH to 1.5 QH. The delivery is regulated by an automatic gate valve. In the figure 2

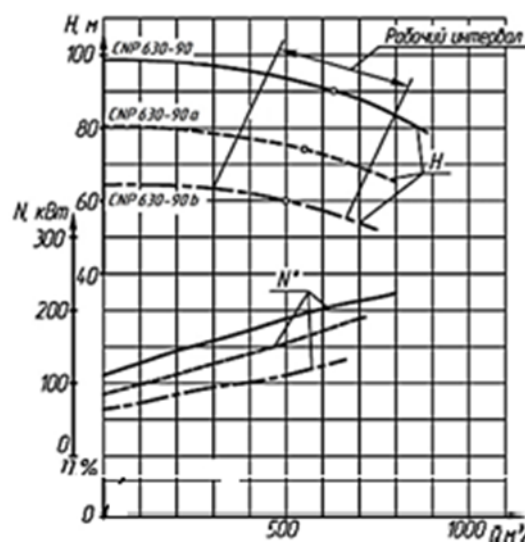


Figure 2. Characteristics of the CNP 630-90 pump unit

To conduct a numerical study of the flow structure in the flowing part of the RC for various methods of head increase using the licensed software product CRADLE/Sflow university version. This software product is based on the method of numerical solution of the fundamental laws of hydromechanics [11] of the equations of motion of a viscous fluid

together with the continuity equation. This is a sufficient condition for the validity of the application of the results of the numerical study. The flow was calculated by numerical solution of the system of equations describing the most general case of fluid medium motion - the Navier-Stokes equations (1) and the continuity equation (2) [12]. The equations are presented in the reduced form (i, j = 1-3), summation by the same indices is assumed

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + f_i, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0, \quad (2)$$

where x_i, x_j - are coordinate axes;

f_i - component expressing the action of mass forces

As boundary conditions, the condition of "sticking" on solid walls (velocity equals zero), distribution of all velocity components in the inlet section, and equality to zero of the first derivatives (in the direction of flow) of the velocity components in the initial section are set. For closure of the Reynolds equations in CRADLE/Sflow, a number of turbulence models are used. A complete list of the capabilities of this software product, its mathematical apparatus and basic hydrodynamics models can be found in the documentation for this software product. The SST turbulence model is best suited for such calculations, and several problems have been calculated using k-g and k-y turbulence models. All of these turbulence models support the method of near-wall functions, which relate the flow parameters to the distance from the wall. This avoids the use of very fine meshes near the wall. The flow was calculated in a stationary formulation. The working medium (water under normal conditions) was assumed to be incompressible and the flow regime was assumed to be turbulent [13].

The following assumptions were made during the numerical study:

- the flow at the inlet to the design region is axisymmetric;
- there is no influence of leakage through the RK seals on the flow in the flow section

3 Method to Modify the Impeller

The concept of variable pitch (Figure 3) is based on the ideas of redistributing predominantly impact losses at the inlet of the impeller blade system throughout the impeller.

The realisation of such solutions implies the use of new design approaches. In particular, the vane system of the pump is calculated not for one point (Q_{acc}, H_{acc}), but for the required flow area ($Q(Q_1, Q_2 \dots Q_n), H(H_1, H_2 \dots H_n)$).

Thus formed hydrodynamic grid has different angles of inclination and width of channels. As a consequence, the working area of the pump is widened.

Thus the average integral efficiency of the pump with heterogeneous vane system is higher than that of analogues with classical (homogeneous) vane system.

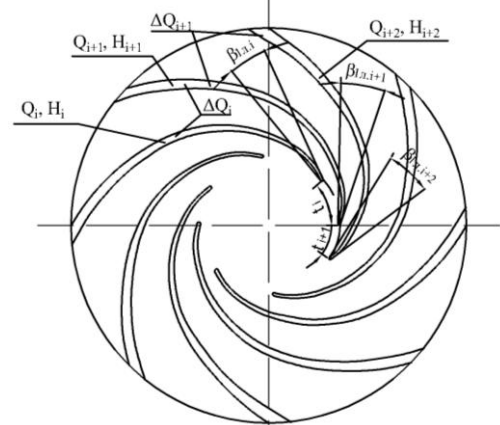


Figure 3. Scheme for calculation of heterogeneous grid parameters

Shock losses in the pump wheel are determined according to the method [14,15]

According to for the projections of absolute velocities on the circumferential direction on the angle of flow and the circumferential direction on the structural angle of the blade we write down:

- projection of absolute velocity on the circumferential direction by the flow angle:

$$h_{y_{d,h}} = \varphi_h \frac{\Delta v_{uH1}^2}{2g}, \quad (3)$$

where:

$h_{y_{d,h}}$ – flow impact losses at the wheel inlet;

φ_h – impact loss coefficient of the impeller grille

Δv_{uH1}^2 – difference of circumferential components of absolute velocity at deviations of flow direction from its direction in the shockless entry mode.

For the projections of absolute velocities on the circumferential direction by the flow angle and the circumferential direction by the blade design angle we write:

- absolute velocity projection on the circumferential direction along the flow angle:

$$v_{uH1}^2 = U_{1H} - \frac{ctg(\beta_{1H})}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} Q; \quad (4)$$

where:

U_{1H} – circumferential speed;

β_{1H} – flow angle on the blade;

R_{H1} – wheel entry radius;

b_{H1} – inlet wheel channel width;

Ψ_{1H} – flow restriction factor;

- projection of the absolute velocity onto the circumferential direction along the grating angle:

$$v_{uH1\pi}^2 = U_{1H} - \frac{ctg(\beta_{1H\pi})}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} Q; \quad (5)$$

where:

$\beta_{1H\pi}$ – angle of the flow on the blade along the grate angle

At a flow rate corresponding to a shockless entry to the wheel, the difference in the circumferential velocity components at the entrance to the NC:

$$\Delta v_{uH1}^0 = v_{uH1}^0 - v_{uH1L}^0 = \left(\frac{\text{ctg}(\beta_{1H}^0)}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} - \text{ctg}(\beta_{1HL}) \right) 2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H} Q_0 \quad (6)$$

where:

Q_0 - constant flow rate for fixed torque values

In Figure 4, consider the following flow model in velocity triangles:

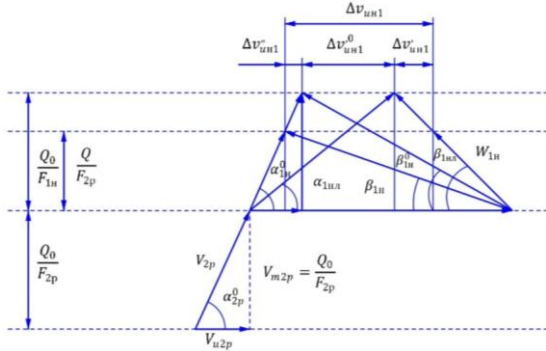


Figure 4. Flow model in velocity triangles for the RK-NK system

It can be seen that the total difference consists of three terms:

$$\Delta v_{uH1}^2 \Delta v'_{uH1} + \Delta v''_{uH1} + \Delta v_{uH1}^0 \quad (7)$$

We have:

$$\Delta v'_{uH1} = \frac{\text{ctg}(\beta_{1H})}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} (Q_0 - Q); \quad (8)$$

$$\Delta v''_{uH1} = \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} (Q - Q_0) \quad (9)$$

Given the ratio

$$v_{uH1}^2 = \frac{R_{p2}}{R_{H1}} v_{up2} = \frac{R_{p2}}{R_{H1}} \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{p2} \cdot b_{p2} \cdot \Psi_{2p}} Q = \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} Q \quad (10)$$

We get:

$$\begin{aligned} \Delta v_{uH1}^2 \Delta v'_{uH1} + \Delta v''_{uH1} + \Delta v_{uH1}^0 = & \frac{\text{ctg}(\beta_{1H})}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} (Q_0 - Q) + \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} (Q - Q_0) + \\ & \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} Q - \frac{\text{ctg}(\beta_{1HL})}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} Q + \\ & \frac{\text{ctg}(\beta_{1H}^0)}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} Q_0 - \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} Q_0 \quad (9) \end{aligned}$$

$$\Delta v_{uH1}^2 = \left(\frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} - \frac{\text{ctg}(\beta_{1HL})}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} \right) Q + \left(\frac{\text{ctg}(\beta_{1H}^0)}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} - \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} \right) Q_0 \quad (11)$$

Denote

$$L = \frac{1}{\sqrt{2g}} \quad (12)$$

$$K = \frac{1}{\sqrt{2g}} + \left(\frac{\text{ctg}(\beta_{1H}^0)}{2\pi \cdot R_{H1} \cdot b_{H1} \cdot \Psi_{1H}} - \frac{\text{ctg}(\alpha_{2pH})}{2\pi \cdot R_{H1} \cdot b_{p2} \cdot \Psi_{2p}} \right) \quad (13)$$

Then it's easy to get:

The method of construction of different blades for the modified wheel was as follows, according to the

data of the range of variation of the feed, according to industrial tests, the heads and geometrical parameters of three blades of the heterogeneous grid were determined from the Q-H characteristics curve (Figure 3), Figure 4 shows the programme windows with the coordinates of the blades.

4. Numerical Method

4.1 Mesh generation

The computational meshes were created using the ICEM CFD mesh generator for their construction. It allows for forced adjustment of the mesh density, densifying it in necessary places (e.g. at the blade inlet and outlet edges) and enlarging it where high mesh density is not required. This approach allows saving computer resources and obtaining sufficient grid density in the investigated part of the computational domain. Before conducting the study, a grid independence check was performed. For this purpose meshes with different densities were constructed.[16] Figure 5 shows the stages of mesh generation.

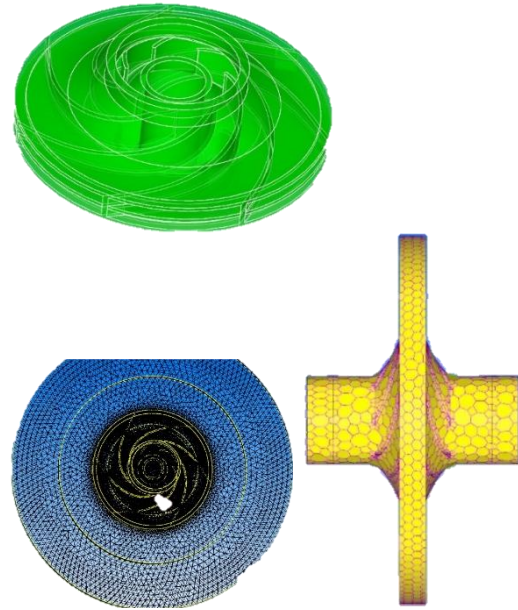


Figure 5. Mesh generation stages

The impeller sub-area was covered with a uniform unstructured mesh of tetrahedral cells, and the outlet sub-area with a regular hexagonal mesh Figure 7. Mesh adaptation was not performed because, firstly, the use of standard wall functions does not require too fine a mesh ($\min y^+ = 30 \div 60$), and secondly, the lack of computational resources prevented further improvement of the solution.

Computational meshes with 600 thousand, 800 thousand and 1mn cells were constructed. The analysis of integral values obtained from the numerical results for meshes with different densities showed that the results differ by no more than 1 % when the number of cells exceeds 700 thousand. This result indicates the mesh independence. Further numerical study was carried out for meshes having ≈ 700 thousand cells. The value of the variable y^+ , which characterises the

densification of the mesh near the walls, was in the range $20 < y^+ < 60$ units, [17].

Boundary conditions were set on the wheel wall - Wall: zero tangential velocity (condition of "sticking" to the wall), zero velocity of the wall itself relative to the rotating coordinate system (i.e., the wall moves synchronously with it). On the outlet wall - Wall: zero tangential velocity (condition of "sticking" to the wall), zero velocity of the wall itself relative to the absolute inertial coordinate system (i.e. the wall is stationary).

4.2 Verification of the algorithm

The above algorithm is used for numerical simulation. The above algorithm is used for numerical simulation of the internal flow of the original centrifugal pump and the optimised centrifugal pump. To check the validity of the algorithm used, calculations were carried out for the working area of the pump according to the experimental data in Figure 6 from $0.5Q$ to $1.25Q$.

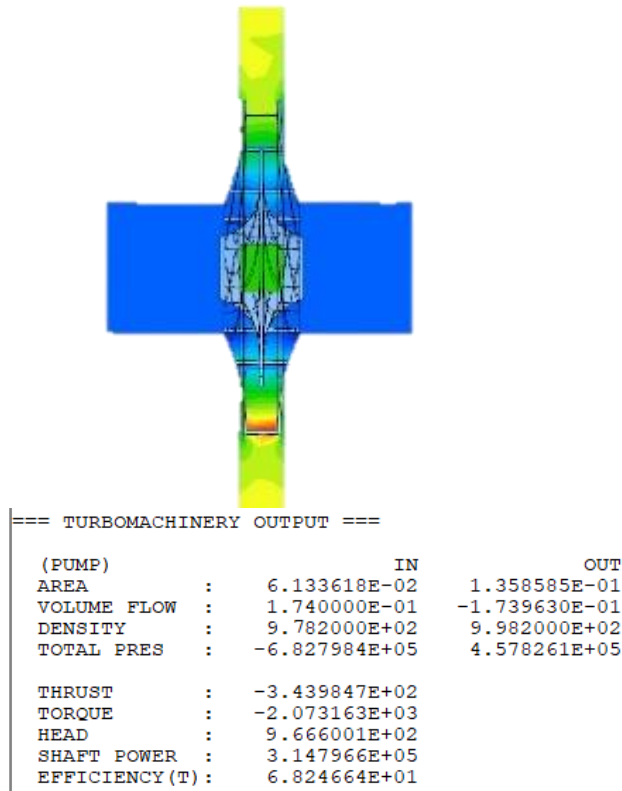


Figure 6. Modelling results a) pressure distribution in the wheel b) file with results

The results of the pressure characteristics of the pump, obtained by calculations in CRADLE/Sflow software and the experimental results from the factory, were summarised in the comparison diagram shown in Figure 7.

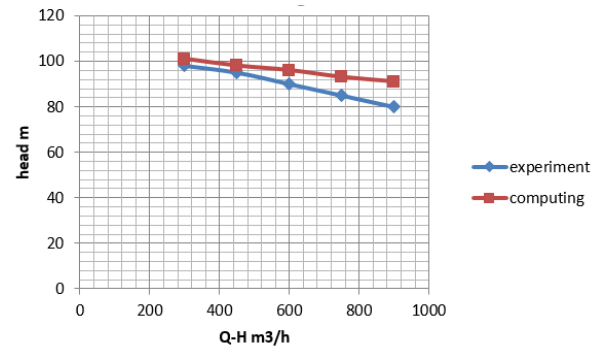
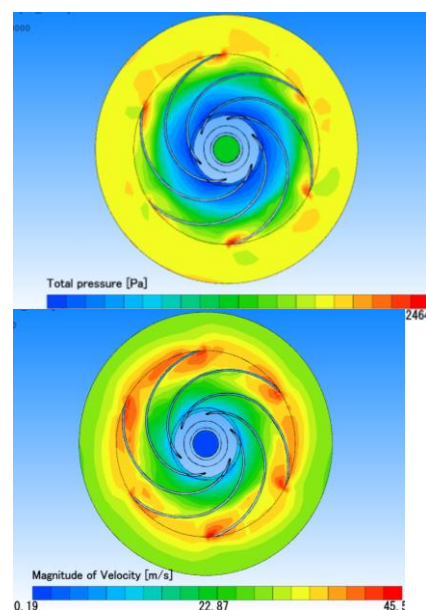


Figure 7. Comparative graph of calculated and experimental values

Analysing the integral characteristics obtained on the experimental bench and comparing them with the results of numerical simulation (Figures 8,9), we can conclude that the discrepancy between the results for head is about 4%. Analysing the fact of mismatch of characteristics, it can be assumed that the most probable reason for this behaviour of curves is the discrepancy between the geometrical model of the vane systems used in calculations and those actually manufactured. In particular, it concerns the shape of the inlet and outlet edges, the roughness of the flow part, etc. as well as the error of physical measurements, but the difference is quite acceptable, which confirms the adequacy of the computer model of the wheel.

5 Numerical Simulation Results

To measure the effect of variable curvature vanes on the performance of a centrifugal pump, the changes in static pressure, relative velocity and turbulence kinetic energy in the middle part of the impeller were analysed under typical flow conditions (low flow rate $0.5Q$ and $0.7Q$, nominal flow rate $1.0Q$ and high flow rate and $1.5Q$).



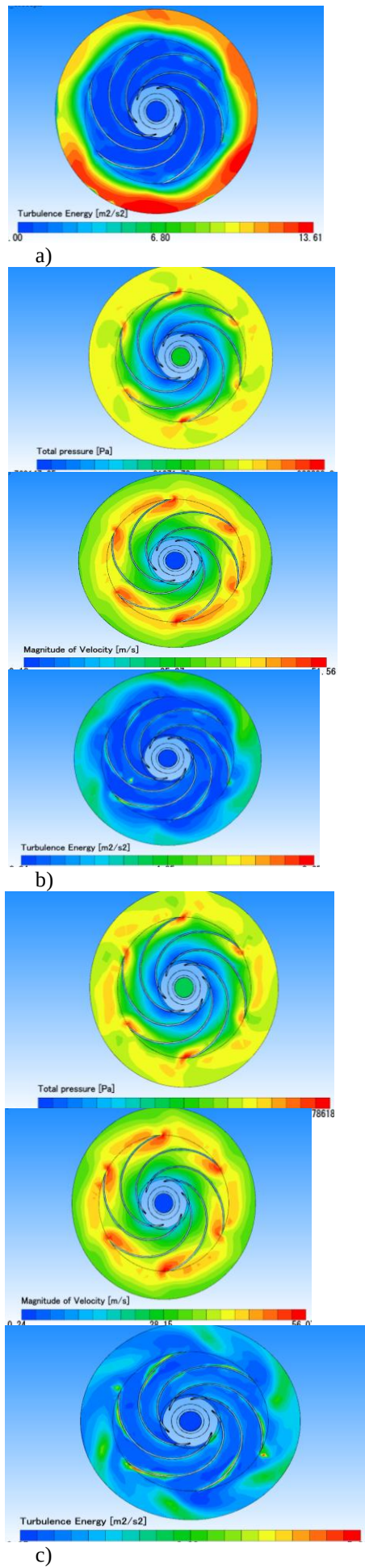
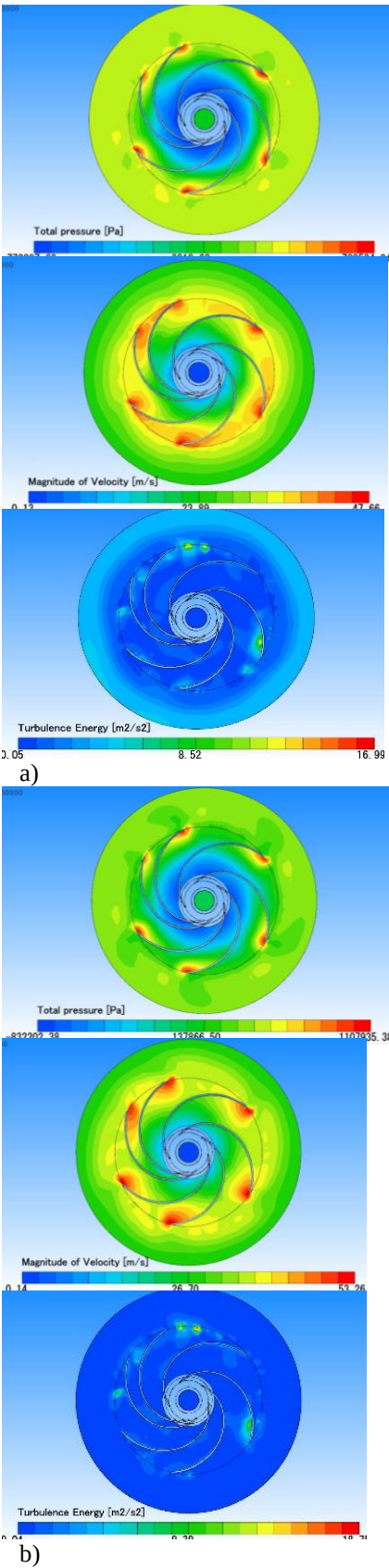


Figure 8. Results of simulation of flow kinematics in a wheel with a homogeneous grid



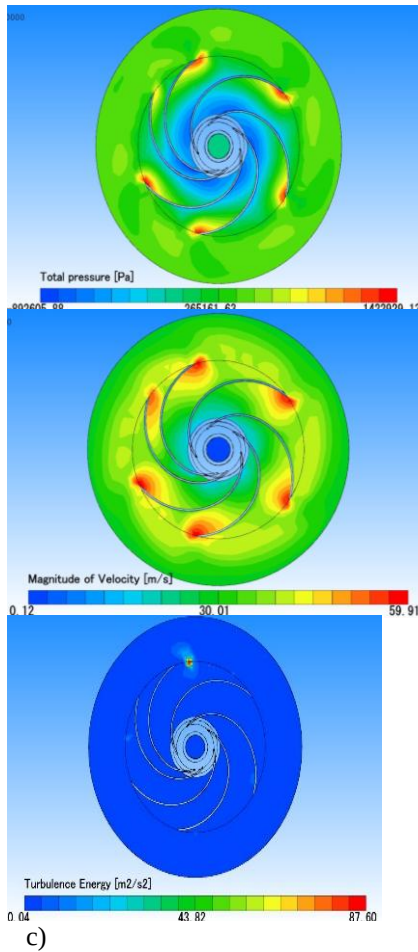


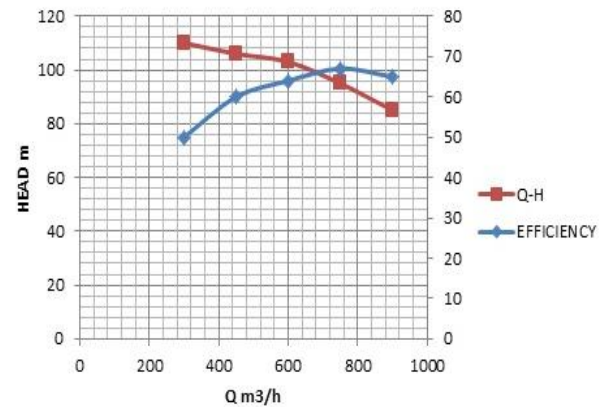
Figure 9. Results of modelling the flow kinematics in a wheel with heterogeneous grating

According to the modelling results, the general law of static pressure variation is that the static pressure in the different channels of the wheel is distributed uniformly. The static pressure drops for the original wheel are 0.36MPa to 0.528MPa and for the wheel with blades of variable curvature from 0.44MPa to 0.63MPa at all investigated feed modes. as shown in Figure 11. The static pressure was lowest at the inlet of the impeller. The static pressure at the impeller outlet was the highest. For all impellers, the static pressure generally increased with increasing flow rate. Under the condition of 0.7 Q, the increase in static pressure was more pronounced. At the outlet, the static pressure reached the maximum value at condition 1.5 Q. Visualisation of the pressure distribution gradient shows a decrease in the underpressure zones in the modified wheel, i.e. a decrease in the occurrence of reverse currents in the wheel with heterogeneous lattice. The volume of the distribution gradually decreased as the flow velocity increased.

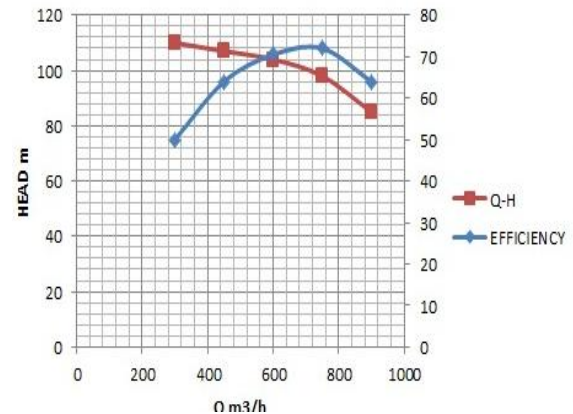
Velocity diagrams allowed us to observe the increase of relative velocity in the modernised wheel, so at minimum feed rate the velocity in the original wheel is 45m/s in the modernised wheel 47m/s and at maximum feed rate 56m/s for the original wheel and 59m/s for the modernised wheel, the increase of relative velocity will affect the hydraulic efficiency.

The distribution diagram of turbulent kinetic energy at low flow velocity in the original wheel has a large surface coloured blue, which indicates a higher value of turbulence and behind the wheel are seen pronounced red areas of maximum values of vortices, which naturally leads to a decrease in efficiency in this mode of operation of the pump. Whereas in the modified wheel all areas of turbulence are coloured blue, and only locally appear vortices. Pump losses at low flow rates are higher in the similar impeller.

According to the results of efficiency and pressure characteristics obtained by the method of computational fluid dynamics in CRADLE system, graphs were plotted (Figure 10).



a)



b)

Figure 10. Pressure and energy characteristics of the pump (a) for the pump with the original impeller (b) for the pump with the modified impeller

In the diagrams, the intersection of the Q-H curve and the efficiency curves, which were plotted according to the results of calculations, correspond to the operating points of the pump. Obviously, despite the fact that the efficiency in the modified pump has decreased at rated flow, the operating point and the average integral efficiency has increased by 2%, but the operating point has shifted towards a decrease in flow by 1.5%.

Conclusions

In this paper, the effect of retrofitting the vane system of a centrifugal pump wheel on the pressure-energy characteristics was investigated. Numerical simulations of flows in a centrifugal pump with homogeneous and heterogeneous vane wheels were carried out at low flow, nominal flow and high flow.

Changes in static pressure, relative velocity, and turbulence kinetic energy were analysed.

The theory of the variable curvature vane system was verified by computational methods.

The head and efficiency of a centrifugal pump with different vane arrays were compared.

The main conclusions are that the static pressure and relative velocity have increased in the modernised wheel, leading to an increase in hydraulic and overall efficiency, the kinetic energy of turbulence has less pulsation, the operating point of the pump has shifted. In spite of the fact that the average integral efficiency has increased in the modified pump, the operating point of the pump is shifted towards a decrease in flow by 1.5%. The theory of the vane system with variable curvature has been verified by computational methods.

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Archiving of structural solutions by task based conceptual design method

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Abstract. The composition of the design libraries and database is essential for finding the optimal structural solution for a task-based conceptual design project. Nowadays the broad usage of digital technologies for the database composition and its update draws on the front line the demand for creating compatible formats for synthesizing and archiving procedures, facilitating the easy and task-based access to the database for the search of the needed solution. It is practical and efficient to organize this process in a task-based manner oriented to a specific group of solutions combined by the field of application and the specific types of tasks. As a result, the general tools of the suggested task-based design method are generalized and updated to the usage of design fragments, which could be general and satisfy for the specific groups of products with their specific properties. The methods and action steps of the named task-based design method are updated accordingly for such a purpose. The method is explained for a large group of locking pliers, showing the general set of properties and the potential design fragments used in the structure of various structural types of these tools.

1 Introduction

The value of any mechanical design is not only in the satisfaction of a design task, development of a product and satisfying the customer needs but also in its ability to accumulate knowledge to ensure recorded input in databases and archives, thus stimulating the further organized usage of the classified data. The procedural success and effectiveness of archiving problems are mostly separated from the development task itself. So far, the developer being responsible for the short-term implementation of the project may care less about database creation and archiving issues, leaving those duty on the side of data recorders or potential users, requiring them not tiny efforts to modify the available data to format subject for further application per designers needs. Considering the circumstance mentioned above from the point of view of a specific conceptual design methodology one can connect the core method of each approach to the development of systemized database, thus evaluating the archiving capabilities of each method. The popular approach of combinatorial search of structural solutions pioneered by and further developed and popularized by [1] emphasize the search of a challenged structural solution among numerously generated topologically abstracted mechanisms independently from functional purposes. Various such topologies compose the appropriate contents of a database which are helpful for the designer for their simplicity and abstract manner; however, they still need the comprehensive functional component necessary for task-based design.

Other design approaches [2,3] specific to the application of task-based block solutions support the formation of task based block databases. The structural blocks being the units of concept design are subject of application as a whole. They hide the inner procedure of their formation, which is the guideline of a specific, even known mechanism formation and its reinvention upon task-based needs.

Another engine [4] for the search of structural conceptual solutions – the theory of inventive problem solving – provides a flexible though limited synthesizing tool set for possible functions and designers. This approach is still lacking in structural solution results archiving in a systemized task-based manner.

All those methods help implement these tasks just from the point of view of those methods' strengths and conceptual essence and if the primary objective is just one single task and a single topology.

A task-based conceptual design method[5], suggested for more than a decade before, implies the development of brand new structural solutions based on a task-based manner of structure composition, where the links or structural groups are connected in an attempt to implement some task or group of tasks, and where the result may be developed independently from the existence of a specific mechanism. However, the development path brings to the formation of a brand new or a known solution, thus shortening the time on the

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search for combinatorial solutions and getting a result satisfying the set of design tasks or suggesting a reasonable tradeoff solution.

So the advantage of the task-based method or the difference against the methods mentioned above is the task-based nature, which means that any action is done in a task-based manner, the simplicity and unification of the set of design tools, the general concept of the simplification of the design environment (the possibility of shaping the sizes of the design environment upon the user's needs and convenience), according to the objective the designer is going to implement, the variety of the formalization, abstraction and presentation tools. As a confirmation of feasibility, workability, or validation of the technique, it is serving dozens of internationally recognized patents for products developed using the task-based design method.

The strict task based manner of the suggested conceptual design method implies involvement of needed sets of tasks for every synthesizing action, thus enables to provide consideration of any essential combination tasks and appropriate structures for their implementation, ensuring the capability to fix the core properties of the mechanism in a couple of the structure, responsible for it. Such an approach serves as the base for creating model solutions specific to the product profile and accumulating such data in a database.

Tasks & objectives:

- Formalize the concept design method using graph symbols.
- Update the method by separating a model based fragment of the design scenario, defining the further progress of concept design.
- Show the developed example for structures of three different lock pliers.
- Provide the composition of the fragments.
- Show the creation of various lock pliers using the developed techniques and following the different design scenarios based on the same model.
- Create the model development following the steps and techniques for implementing the task-based design method.

2. Task-based design and archiving problem

2.1 The idea of conceptual design

The idea of the suggested method of conceptual design is in direct dependence (1) of the mechanical structure and the tasks the mechanical structure is intended to perform:

$$A_{mn} = f(T_{ij}) \quad (1)$$

where: A_{mn} is the contents of the mechanical structure (the mechanism),

T_{ij} is the set of tasks challenged against the mechanical structure.

Double indexing is for components of the both sets is for indication of the component itself ($m = n, i = j$) and for indication of a relation between the components ($m \neq n, i \neq j$).

2.2 Graph presentation of the idea

A graph G presentation of the mechanical structure shows the contents $G = (V, E)$ contents

A graph presentation of the mechanical structure $G = (V, E)$ contains the set of links V as the vertices of the graph and the set of edges E as connections between them(2).

$$G = (V, E) = (V, E_i, E_k) \quad (2)$$

The edge E is duplicated by two parallel connected edges of E_i for challenged properties or tasks and by E_k for kinematical links and other types of relations (for examples shape forming movement of a technological tool relative the surface of a workpiece subject to processing). Those two edges are representing the set of tasks the future mechanism should perform. The first edge (E_i) should indicate the design plan (task), and the second edge (E_k) should provide the proper type of connection to implement the given task.

The first edge E_i complies to the conditions (3).

$$E_i = (t_{12}, t_{23}, \dots, t_{ij}) = A'_{mn} = T_{ij}, i \neq j \quad (3)$$

where $t_{12}, t_{23}, \dots, t_{ij}$, are the tasks of the a task set T_{ij} , corresponding to the set of edges of mechanism set A'_{mn} .

The second edge E_k edge complies to the conditions (4).

$$E_k = A_{mn} = (a_{12}, a_{23}, \dots, a_{mn}), m \neq n \quad (4)$$

where $a_{12}, a_{23}, \dots, a_{mn}$, are the links of the mechanical set combined in the mechanism set A_{mn} .

The mechanical set itself consists of links (5)

$$V = A_{mn} = (a_{11}, a_{22}, \dots, a_{mn}), m = n \quad (5)$$

where $a_{11}, a_{22}, \dots, a_{mn}$ are the links. As mentioned above, a link a_{mn} is double indexed for indication of a link when $m = n$, and for indication of a relationship when $m \neq n$.

Analogously the task set (6) includes the tasks $t_{11}, t_{22}, \dots, t_{ij}$ themselves ($i = j$) and the possibility to translate a task into implementable status of a task ($i \neq j$):

$$E_i = T_{ij} = (t_{11}, t_{22}, \dots, t_{ij}), i = j \quad (6)$$

where $i = j$ corresponds to a specific design task,

$i \neq j$ corresponds to translation operator from a task to an implementable status of a task.

2.3 Conceptual Design Procedure, The Start of Conceptual Design

Firstly, the two major essential a_{11}, a_{22} links of conceptual design are granted and identified (7).

$$A_{mn} = (a_{11}, a_{22}, \dots, a_{mn}) = f(T_{ij}) = f(t_{11}) \quad (7)$$

The purpose of granting the initial two links is to plan an implementation of the initial task t_{11} .

For visualization and topological presentation purposes those links are identified as of vertices of a graph which are connected by two graph edges. The first is symbolizing the mechanical task, those links should perform and the other edge symbolizes the type of mechanical relation a_{12} (kinematic joint in particular) which is responsible for implementation of the above-mentioned task or function.

2.4 Expansion and Squeezing of Both Sets

Further development of mechanical components and task sets is organized and carried out through expansions and further necessary squeezing of the mentioned sets within step-by-step judgments and evaluation of the results. The expansion \ squeezing procedures are formalized by the addition or subtraction of unit links and tasks to the original contents of both sets.

Expansion of a mechanical set means adding a link or a structural group to the existing state of the set in a task-based manner that is challenging to the satisfaction of a task or group of tasks (8).

$$\begin{aligned} 1, l \leq m, n \leq M \text{ +/- } S, N \text{ +/- } S \\ \text{where } S=1 \text{ for a single link} \\ S=s \text{ for a structural group} \end{aligned} \quad (8)$$

Expansion of the task set means translating an initial task into another task subject to implementation and adding new tasks as challenging ones generated due to implementation of the previous tasks (9).

$$\begin{aligned} 1, l \leq i, j \leq I \text{ +/- } D, J \text{ +/- } D \\ \text{where } D=1 \text{ for a single task} \\ D=d \text{ for a group of tasks} \end{aligned} \quad (9)$$

If the link set expansion is enough or satisfies all the current tasks, the new challenging tasks are considered, and the task set is expanded or squeezed analogously. The procedure is continued until all the tasks are satisfied by the updated set of links.

The expanded state of the both sets - formalized by (1) with consideration of set modification procedures (8) and (9) - includes vaster combinatorial possibilities to proceed with the optional structural solution. For organizing this phase of conceptual design specific portions of both sets are limited by the number of components - not more than 5 for each set - isolated and considered separately. Isolation of a subset from both sets is formalized in (10) and (11). Adding single links to the indexes of the units of the mechanical components set creates a maneuvering frontier providing satisfaction of a specific design task.

2.5 Models and local design scenarios

When considering a limited set of mechanical components and design tasks, a practical judgment and evaluation of a task's best synthesis result can be reached. The number of components in mechanical and task sets should be 5 or 6 for easy acceptance and manageability. Those fragments play a significant and crucial role in limiting and visualizing the design task and the mechanical components responsible for those tasks.

$$g[A_{mn}] \sqsubseteq A_{mn} \quad (10)$$

$$g[T_{ij}] \sqsubseteq T_{ij} \quad (11)$$

Subsets formed per (10) and (11) are the models or decisive design fragments playing guiding role for the further conceptual design process. For evaluation of their effectiveness, they are evaluated against satisfaction the specific set of functions (12).

$$\begin{aligned} Ev[G] = (V, E_k, E_t) = e \\ \text{If } e=1 \text{ the task is implemented,} \\ \text{if } e=0 \text{ the task is not implemented} \end{aligned} \quad (12)$$

For getting those limited components subsets the component squeezing actions are implemented against for major sets and mutually dependent two subsets of mechanical components and task components are formed: $g[A_{mn}]$ and $g[T_{ij}]$. When applied for synthesis purposes the procedure is run by addition of links and tasks while in the case of product analyzes the components are combined in decisive subsets leading to the implementation of the core function of the product and development and satisfaction of the further supportive functions per conceptual design needs.

For the products having the same or similar functions that's interesting to anchor to the subset for mechanical and task components which are defining the further product development scenario. Such cross point of design scenario can be considered as the model or substance structure of the product making it similar to other products per implementation of the main task and making it different from the similar products by uniqueness of the further path of structure development.

3. Fragment based analyses of a variety of locking pliers

The backwards order of the procedure described for synthesis actions, that is, revealing the conceptual structure inside the physical prototype or physical description of locking pliers invented for the last 60-plus

Firstly, the feature of adjustable jaw capacity is searched, classified and separated from the whole set of properties a locking pliers needs to have.

Thirdly, the kinematical diagram is further processed into graph formats with both kinematical joints indications of the angular and translational movement parameters, adding the edges symbolizing the tasks and properties they perform.

Fourthly, separating and finding the crossways leads to different structural solutions. After confirming the challenged property of adjustable jaws, the rest of the listed properties are provided one by one by the involvement of necessary links and/or structural links for providing single or grouped properties of the locking pliers. Once the whole set of actions of conceptual design is completed, the different structural solutions of locking pliers can be numerically evaluated, as summarized in Tables 1,2,3.

Table 1. Conceptual solution per Pat.2,768,549

Table 1. Conceptual solution per Pat.2,768,549

TOGGLE-CONTROLLED PIVOTED JAW WRENCH

US Patent 2,768,549 - 1956

A combined plier and wrench comprising a handle having a U-shape in transverse section, a fixed jaw at the forward end of said handle, a movable jaw, said fixed jaw being disposed at an oblique angle relative to the length of said handle, guide means carried by said handle disposed at a right angle relative to the gripping face of said fixed jaw, an elongated jaw adjusting bar disposed in said handle, a pair of slide plates fixed to the forward end of said bar and engaging said guide means, means pivotally securing said slide splats to said movable jaw, means for adjusting the lengthwise position of said bar relative to said handle, a link pivotally connected at one end to the rear portion of said bar, a second handle pivotally secured at its forward end to said movable jaw, means pivotally securing the forward end of said link to an intermediate portion of said second handle whereby in the closed position of said second handle said latter pivot means will be disposed inwardly of the plane between the end axes of said link, to thereby lock said second handle and movable jaw in gripping position, parallel with the gripping face of said fixed jaw, and a spring connected between said first named handle and said movable jaw.

Physical Diagram

Kinematical Diagram

Translated Function Set

t ₁	Perform lock function	t ₁₁	Provide high trans. ratio
t ₂	Adjustable jaw range	t ₂₁	Limited jaw range
t ₃	Jaws parallelism	t ₃₁	Strictly parallel jaws
t ₄	Convenient usage	t ₄₁	Natural hand position
t ₅	Fixation, adjustable force	t ₅₁	Irreversible stop
t ₆	Self-operation	t ₆₁	Assisted open - close

Translated Link Set					
10	Elongated Handle	a_1	28	Link	a_4
10	Fixed Jaw Member	a_1	33	Adjustment Screw	a_5
15	Clamping Member	a_2	31	Bar	a_6
26	Handle	a_3	27	Coil Spring	a_7
Structural-functional subsets					
1			$A_{mn} = (a_{11}, a_{22}) = f(t_{11})$		
2			$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}) = f(t_{11})$		
3			$A_{mn} = [a_{11}, (a_{22} \dots a_{44}), a_{66}] = f(t_{11}, t_{21})$		
3		$A_{mn} = [a_{11}, (a_{22} \dots a_{44}), a_{66}] = f(t_{11}, t_{21})$			
4			$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}, a_{66}) = f(t_{11}, t_{21})$		
5			$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}, a_{66}, a_{55}) = f(t_{11}, t_{51})$		
6			$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}, a_{66}, a_{55}, a_7) = f(t_{11}, t_{51}, t_{61})$		

Table 2. Conceptual solution per Pat.3,672,245

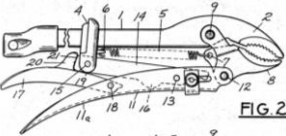
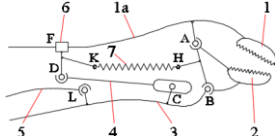
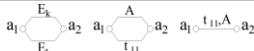
Table 2. Conceptual solution per Pat.3,672,245

WRENCH US Patent 3,672,245 – 1972					
Pliers for positively gripping a selected work piece between substantially parallel gripping jaw surfaces, one of which is carried by a fixed handle, the other of which is carried by a pivoted toggle member mounted on an extension of the fixed handle for pivotal movement and adjustment toward and away from the jaw on the fixed handle, a handle pivotally connected to the toggle member and tensioning means connecting said handles and adjustment means on the fixed handle and having pivotal connection with the pivoted handle whereby the gripping effort on a selected work piece may be adjusted to secure a strong positive grip thereon without causing damage to the work piece.					
Physical Diagram			Kinematical Diagram		
Translated Function Set					
t ₁	Perform lock function	t ₁₁	Provide high trans. ratio		
t ₂	Adjustable jaw range	t ₂₁	Limited jaw range		
t ₃	Jaws parallelism	t ₃₁	Strictly parallel jaws		
t ₄	Convenient usage	t ₄₁	Natural hand position		
t ₅	Fixation, adjustable force	t ₅₁	Irreversible stop		
t ₆	Self-operation	t ₆₁	Assisted open - close		
Translated Link Set					
2	Jaw Portion	a ₁	5	Channeled Handle	a ₃
14	Arcuate Rib	a _{1a}	4	Toggled Arm	a ₄
1	Fixed Handle	a _{1b}	3	Adjust. Screw	a ₅
8	Jaw	a ₂	6	Spring	a ₆

Structural-functional subsets		
1		$A_{mn} = (a_{11}, a_{22}) = f(t_{11})$
2		$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}) = f(t_{11})$
3		$A_{mn} = [a_{11}, (a_{22} \dots a_{44}), a_{1a}] = f(t_{11}, t_{21})$
4		$A_{mn} = (a_{11}, a_{1a}, a_{22}, a_{33}, a_{44}, a_{55}) = f(t_{11}, t_{51})$
5		$A_{mn} = (a_{11}, a_{1a}, a_{22}, a_{33}, a_{44}, a_{55}, a_{66}) = f(t_{11}, t_{51}, t_{61})$

3.3 Design scenario per Pat. 3,545,316

Table 3. Conceptual solution per Pat. 3,545,316

PLIERS					
US Patent 3,545,316 - 1970					
In a plier type toggle wrench having a handle with a stationary jaw at its front end, a movable jaw pivoted to said front end of the handle, toggle means for forcing the 45 movable jaw toward the stationary jaw including an actuating lever having one end pivoted to the movable jaw and the other end with a hand grip section spaced from and opposite the back end of the handle, an adjustable abutment on the handle, a toggle link having one end pivoted 50 to the abutment and the other end pivoted to the actuating lever between its ends, the movable jaw being forced toward the stationary jaw by squeezing the hand grip section of the actuating lever toward the handle to bring the common pivot between the actuating lever and the toggle link into dead center relation with the other pivots, a release lever pivoted on the actuating lever for prying the hand grip section of the actuating lever away from, the handle to bring said common pivot to the release side of dead center remote from the handle, said release lever 60 having a catch engaging a cooperating catch element in substantially fixed relation to the handle as the common pivot moves to said release side of dead center to arrest the momentum of the toggle means.					
Physical Diagram		Kinematical Diagram			
					
Translated Function Set					
t ₁	Perform lock function	t ₁₁	Provide high trans. ratio		
t ₂	Adjustable jaw range	t ₂₁	Average jaw range		
t ₃	Jaws parallelism	t ₃₁	Strictly parallel jaws		
t ₄	Convenient usage	t ₄₁	Natural hand position		
t ₅	Fixation, adjustable force	t ₅₁	Irreversible stop		
t ₆	Self-operation	t ₆₁	Assisted open - close		
Translated Link Set					
1	Handle	a _{1a}	14	Link	a ₄
2	Fixed Jaw	a ₁	20	Catch	a ₅
3	Movable Jaw	a ₂	4	Tilttable Slide	a ₆
11	Actuating Lever	a ₃	5	Tension Spring	a ₇
Structural-functional subsets					
1					$A_{mn} = (a_{11}, a_{22}) = f(t_{11})$

2		$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}) = f(t_{11})$
3		$A_{mn} = [a_{11}, (a_{22} \dots a_{44}), a_{1a}] = f(t_{11}, t_{21})$
3a		
4		$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}, a_{66}) = f(t_{11}, t_{21})$
5		$A_{mn} = (a_{11}, a_{22}, a_{33}, a_{44}, a_{66}, a_{55}) = f(t_{11}, t_{21}, t_{61})$

3.4 Three archived results for the adjustable jaw feature of locking pliers

The jaw based on the analyses above the following fragments of design scenarios are pointed out which are defining the further and final features of the synthesized solutions as described below.

Table 4. Design fragment per Patent 2,768,549.

	$A_{mn} = [a_{11}, (a_{22} \dots a_{44}), a_{66}] = f(t_{11}, t_{21})$	
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The jaw position adjustability model leads to inconveniency of adjustment action, necessity of two hand application, absence of self-adjustment and self-regulation and strictly to not parallelism of the jaws.

Table 5. Design fragment per Patent 3,672,245.

3		$A_{mn} = [a_{11}, (a_{22} \dots a_{44}), a_{1a}] = f(t_{11}, t_{21})$	
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The jaw position adjustability model leads to a large range of jaw adjustment capacity, the necessity of two hand operation, slower adjustment action, and further tool adjustment for the locking force after the jaw adjustment is made.

Table 6. Design fragment per Patent 3,545,316

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The jaw position adjustability model leads to not parallelism of the jaw flats, slower and not self-adjustment for varying the distance between the jaws, excessive gaps and presence of idler movements in the structure, due to the tiltable sliding member.

Conclusions

1. The archiving properties and potential of the conceptual design methods are projected by their core design features and procedural steps.
2. The procedural steps of the suggested task-based design method serve archiving purposes smoothly and allow the isolated design fragments in the entire design process, which are responsible for a chosen main property or feature of the product, as well as tracking the formation of the remaining critical features.
3. The design procedure applies to the examples of locking pliers providing the possible mechanism reinvention path leading to the composition of the final structure and structuring an intermediate solution as a challenged design fragment.
4. As a result, three different design paths for three locking pliers of different years are composed, explaining the essence of the mentioned locking pliers properties.
5. A graph representation of a task-based design problem unifying the contents of both mechanical and task components in a single graph, including the actions for modifying the contents of both sets and the set and types of kinematical joints of the mechanical set.

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Hinge-lever mechanisms of variable structure and their applications in engineering

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Abstract. The report discusses the transition of hinge-lever mechanisms from one structure to another and the use of these properties to create machines for various purposes. Our research has shown that when some parameters or their ratios change, conditions are created for a possible change in the structure of the mechanism. One of the schemes of hinge-lever mechanisms of variable structure was used in the design of the drilling module as part of the Luna-24 station for drilling a hole on the surface of the Moon, as a generator of power pulses. A major application of lever mechanisms of variable structure was their use as a working body in pressure presses. The report outlines the stages of creating impact machines based on mechanisms of variable structure, from hand-driven perforators to hammers with high impact energy. The design research was accompanied by studies of the kinematics and dynamics of impact machines with mechanisms of variable structure, the identification of new properties, the synthesis of the most optimal design and operating parameters of the power transmission.

Hinge-lever mechanisms, if we keep in mind the modern definition of the concept of mechanism, are among the very first mechanisms that were used by people to facilitate their physical labor. Most of the mechanisms used in the technique are hinge-lever mechanisms, the most famous of them are: crank-slider, crank-rocker, two-crank, two-crank, crank-rocker, sinus, tangential. These mechanisms are used both separately, as executive bodies of various machines, and together as part of technological machines. Due to the widespread use of hinge-lever mechanisms, their structure, kinematics and dynamics of machines based on them are quite well studied. However, in the course of research on the possibility of creating impact-rotational machines for drilling in hard-to-reach places, new properties of these mechanisms have been revealed. It was found that under certain conditions, namely, when some parameters or their ratio change, both the structure of the hinge-lever mechanisms, their appearance, and the laws of motion of individual links, as well as the number of movable links, change. Such mechanisms are called variable structure mechanisms.

In the 1970s, the Academy of Sciences of Kyrgyzstan began work on the creation of a drilling machine, which, as part of the Luna-24 station, was intended for drilling holes in order to extract the core without disturbing the stratification of the lunar soil (Figure 1). One of the main components of the drilling machine was a hinge-lever mechanism that transformed the rotational motion of the engine into the translational motion of the impact mechanism. As such a mechanism, a pulse generator with a hinge-lever firing pin was used [1]. The mechanical pulse generator during the acceleration of the striker is a five-link crank-rocker mechanism (Figure 2, a), which during the cocking of the striker works as a crank-slider mechanism (Figure 2, b).

To switch from one type of mechanism to another, it is necessary to change one of its geometric

parameters, namely the distance between the crank rotation supports and the rocker arm, since changing this distance leads to a change in the rocker arm swing angle. In one of the extreme positions, the rocker arm is captured by a slider that serves as a support, as a result, the mechanism switches to a crank-slider mode of operation (Figure 2, b). In the extreme upper position of the slider, there is no moment of forces jamming the rocker arm. The rocker arm is released, the structure of the mechanism changes, the mechanism switches back to the crank-rocker mode (Figure 2, d). Under the action of the accumulated energy of the spring installed between the rocker arm support and the striker, the latter it is moving downwards rapidly. Thus, a change in the structure is necessary for the firing pin to be cocked, followed by the use of compressed spring energy to accelerate it. It should be noted that the service life of the shock unit is limited by the service life of the spring. For work in a limited time, this mode of operation is quite acceptable.



Fig.1. The layout of the Luna-24 station

In connection with the curtailment of the lunar exploration program, we began to think about using the results of research on the creation of a shock mechanism in terrestrial conditions. The most suitable were the presses for metal processing by pressure. In

mechanical engineering, a huge number of crank-slider presses are used, in which couplings are used to turn the executive body. The same

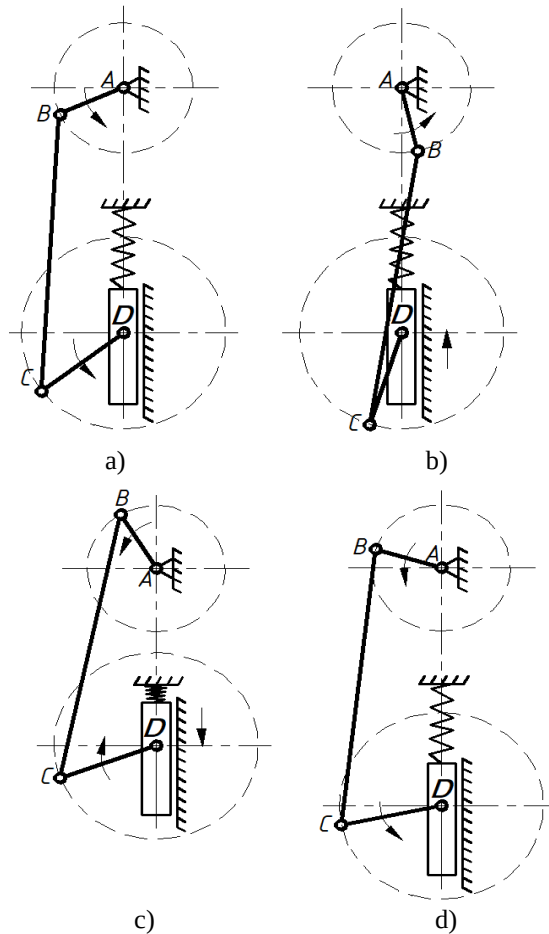


Fig.2. Mechanical generator of power pulses.

couplings limit the speed of the presses, i.e. its performance. However, the specific working conditions of the executive body of crank presses do not allow them to be replaced by a mechanism of variable structure in the form they were used in drilling machines. Therefore, a new scheme of a hinge-lever mechanism of variable structure was proposed [2], in which the mechanism has two stable modes of operation: crank-rocker and crank-slider (Figure 3). In idle mode, the mechanism operates as a crank-rocker mechanism (Figure 3, a), and in operating mode – as a crank-slider (Figure 3, c). The transfer from one mode of operation to another is carried out by changing the position of the rocker arm support mounted on the slider (Figure 3, b), which, in its turn, it is made with the possibility of moving by a small amount. With the help of force action, the inter-support distance of the crank and the rocker arm of the mechanism changes and the rocker arm swing zone changes. In one of the extreme swing positions, the rocker arm closes on the slider, and with further rotation of the crank, the mechanism switches to the crank-slider mode of operation. The reverse transition of the mechanism is carried out by a force action that opens the rocker arm from the slider (Figure 3, d).

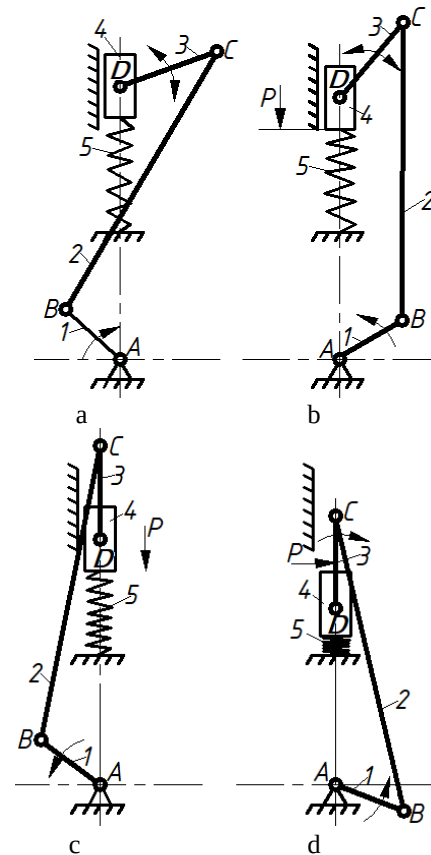


Fig.3. Kinematic diagram of the executive body of the press machine

It should be noted that the use of a fundamentally new technical solution made it possible to create a number of new designs of presses for metal forming, including high-speed automatic presses.

Hinge-lever mechanisms of variable structure are widely used in percussion machines for various purposes, ranging from hand tools to attachments. Research in their kinematics has shown that with certain ratios of the lengths of the links of the hinge-lever mechanisms, the gear ratio from the leading link to the slave reaches large values (Figure 4).

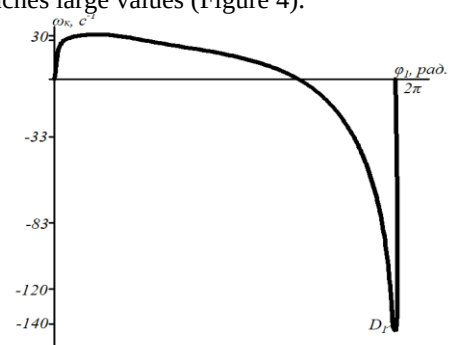


Fig. 4. Dependence of the angular velocity of the rocker arm on the angle of rotation of the crank.

Moreover, the gear ratio reaches the greatest value at the time of the so-called indefinite position of the mechanism, when the links of the mechanism line up. Using this effect, we have developed impact

machines based on mechanisms of variable structure, ranging from hand-driven variable structure, ranging from hand-driven punchers to hammers with high impact energy. Fig. 5 shows one of the schemes of a shock machine with a hinge-lever mechanism of variable structure [4], which implements the pattern shown in Fig. 4.

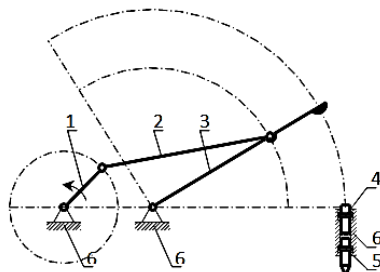


Fig. 5. Schemes of a shock machine with a hinge-lever mechanism of variable structure.

The design studies were accompanied by studies of the kinematics and dynamics of impact machines with variable mechanisms structure, identification of new properties, synthesis of the most optimal design and operating parameters of the power transmission [4, 5]. Mathematical models of these machines have been developed that describe the interaction of the elements of the power transmission: engine, gears, executive body, as well as models of the interaction of the transmission with the treated medium. It should be noted that the conducted experimental studies have confirmed the adequacy of the developed dynamic models and the possibility of their use in the analysis and synthesis of new designs of machines with mechanisms of variable structure.

Here is a fragment of the calculation of the impact node of one of the schemes for the implementation of a hinge-lever mechanism of variable structure – a jackhammer with a two-crank-slider mechanism. Usually, impact machines, during operation, perceive large dynamic loads on the supports, which lead to breakage of the elements of the support unit [3, 4]. New technical solutions were used in this mechanism, which significantly reduce the dynamic loads on the supports. The principle of operation of the mechanism is that when the cranks rotate, the piston moves inside the slider-striker, informing it of translational movement along the axis of the barrel. To reduce dynamic loads in the end sections of the piston stroke, there are air chambers "A" and "B" in the slider-striker, in which air cushions are formed. From the piston, the movement of the slider-striker is transmitted through compressed air formed in the chambers. The presence of an air cushion reduces the resulting dynamic loads when changing the direction of movement of the slider-striker from the working stroke to idle and back. The peculiarity of this mechanism is that at the moment of striking the tool, the slider-striker is kinematically not connected with the piston, i.e. with the drive, which protects the drive elements from the effects of reactive forces.

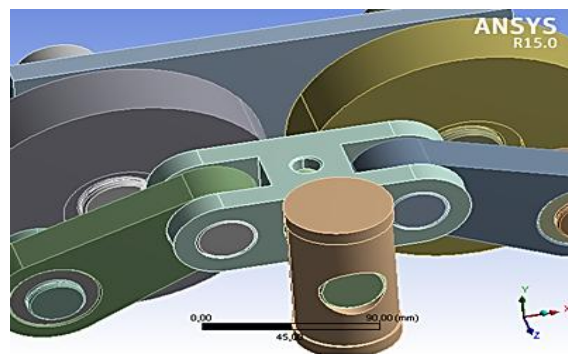


Fig. 6. Hammer impact assembly

To analyze the effect of reactive forces on the support units during the operation of the mechanism and to present a picture of the loaded sections of the shock unit (Figure 6), the stress-strain state of the shock unit was calculated using the computer program ANSYS Workbench 15.0. The calculation results (Figure 7) showed that the greatest stress falls on the bearing units. The minimum value is 0.0004 MPa and it falls on the flywheel zone, and the maximum value is 138.6 MPa, on the bearing assembly connecting the flywheel and connecting rod.

The highest voltage perceived by the shaft connecting the gear wheel and the flywheel is 1.3 MJ ($1 \text{ J} = 10^{-3} \text{ MJ}$). During the life tests of the jackhammer, one of the reasons for the shutdown was the breakdown of the rolling bearing (Figure 8, b) exactly at the place where the program indicates the maximum voltage value. Which shows that the calculated values correspond to the experimental results. The various voltage values are shown in Figure 8, which are highlighted in the corresponding colors. The calculation results showed the probable location of the sliding bearing failure, identified as a result of the theoretical calculation of durability. As can be seen, the results of the calculation and the experiment coincide, which indicates the adequacy of the obtained results of static calculation.

The studies carried out using new methods of modeling and calculating the stress-strain state of structural elements made it possible to identify the nodes that perceive the greatest dynamic loads, as well as to establish the permissible number of work cycles of the most critical links used in the impact mechanism. These results are confirmed by tests of experimental samples of impact machines with mechanisms of variable structure. The ways of increasing the working resources of impact machines with variable structure mechanisms using new technical solutions that significantly reduce the dynamic loads perceived by the support nodes are proposed.

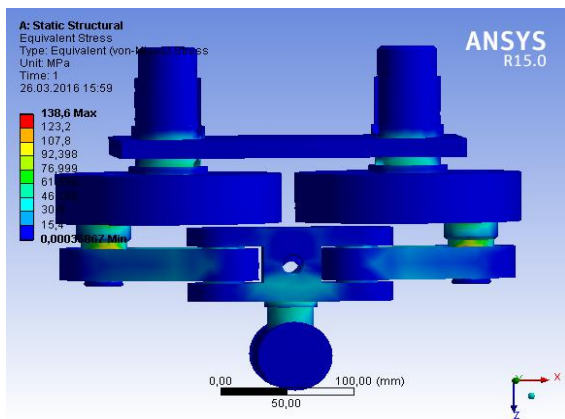
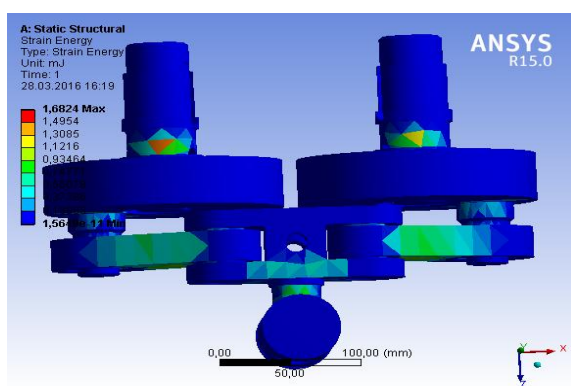
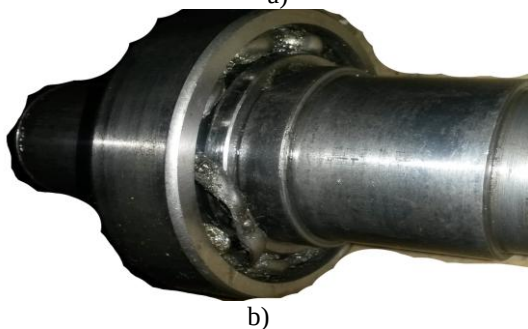


Fig. 7. Results of the analysis of the stress state of the shock node.



a)



b)

Fig.8. Stress state analysis and characteristic rolling bearing failure.

Conclusions. The results of the research have shown that hinge-lever mechanisms of variable structure, acquiring new properties, make it possible to create machines and mechanisms on a fundamentally new basis. The new properties of these mechanisms expand the boundaries of their use, significantly increasing their efficiency. The use of new methods of modeling and calculating the dynamics of machines makes it possible to determine the most loaded places that require increased attention, which in turn contributes to a better design of mechanical impact machines that provide a high degree of reliability and durability.

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Geometry optimisation of highly crowned gear couplings working in high misalignment applications to reduce tooth root stresses

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Abstract. Crowned gear couplings are mechanical components used to transmit power between misaligned rotating shafts. Their geometry is characterised by a significant longitudinal crowning to accommodate angular misalignment. Recent studies reveal that high misalignments drastically reduce the number of teeth in contact and lead to an uneven load distribution among engaged teeth. Consequently, tooth root fracture becomes a common failure mode. Current standards only address misalignment angles below 1.5° , treating applications with greater misalignments as special cases without design guidelines or stress prediction methods. This study proposes a procedure to optimize the design of crowned gear couplings working in high misalignment applications by determining tooth root stress distribution. The geometry is analytically generated, while finite element models are used to calculate the stress distribution. Experimental validation is performed using a dedicated test rig. The obtained results are very close to the ones from the numerical model, demonstrating the suitability of the method for crowned gear couplings operating under significant angular misalignments. The optimized design reduces tooth root stress by 50%, which will increase the fatigue life of the component or enable the application of higher torque values.

1 Introduction

Gear couplings are used to transmit power between shafts due to their high-power density compared to other non-splined connections [1]. Crowned spherical gear couplings are specially designed to work with high misalignment angles ($\gamma \gg 1^\circ$). These couplings require active surfaces with a large amount of longitudinal crowning to obtain a favourable contact pattern under high misalignment conditions ($3^\circ \leq \gamma \leq 10^\circ$). In addition, this longitudinal crowning prevents the teeth from interference with the sleeve during operation and balances the clearance between the hub and sleeve, while increasing the contact ratio [2]. However, the large amount of longitudinal crowning ($\gg 100 \mu\text{m}$), coupled with the small tool path radius used in its manufacture, poses challenges when analytically generating the surfaces of crowned gear couplings. One of the most significant is the appearance of singularities (corresponding to undercut sections). For this reason, an analytical generation model that takes into account the thread surface of the hob during generation has recently been developed [3] and experimentally validated [4]. This model has advantages over other published models [5,6], because it is able to generate tooth surfaces with a large amount of crowning and to produce undercut sections. Moreover, none of the models present in the literature were experimentally validated, as is the case of the proposed model [4].

The gear couplings must have a specific amount of backlash to avoid interference and, at the same time, enable the angular misalignment [6-9]. Angular clearance is the angle of rotation that exists when rotating the teeth on the hub while keeping the sleeve stationary, i.e. from the point where the left flanks are in contact to the point where the right flanks are in contact [7,10]. Since the tooth surface is crowned and there is angular misalignment, the value of the clearance will not be constant for all teeth [2,11]. Indeed, the tilting angular pitch positions will have a smaller clearance than those on the pivoting position, resulting in a progressive contact of the teeth [12]. Thus, the teeth in the tilting position are the first to come into contact and will experience the greatest tooth root stresses [13,14]. In addition, the uneven clearance distribution will cause the number of teeth in contact to vary as a function of the applied torque and misalignment angle, resulting in a variation of the coupling stiffness [15]. To determine the number of teeth in contact, the scientific literature shows finite element models [16,17] and unloaded tooth contact analysis algorithms [3,18]. However, the standards [7,9,19,20] do not include the effect of geometry or loading conditions in their equations. That is, they define the percentage of the number of teeth in contact only as a function of the misalignment angle, without taking into account their stiffness and applied torque. For this reason, it is a common criterion to consider that half of the coupling teeth will be in contact [7-9,19]. This

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criterion may be adequate in applications with small angular misalignments ($\gamma \gg 1^\circ$), however, finite element models [17,21] have shown that it is not extensible for applications with high misalignment. The main reason is that the number of teeth in contact can be even less than the half, especially in low load and high misalignment applications. This fact was demonstrated numerically [17] and experimentally [4].

Since gear couplings are frequently used in applications with small misalignment angles [14,22], most experimental studies and test rigs are oriented towards these working conditions [23,24]. The conducted studies have demonstrated that the contact pattern is not centred nor maintained constant under misalignment conditions [6,25], and wear increases with the misalignment angle [26]. However, Mancuso [27] demonstrated that common failures in couplings operating under heavy applications with up to $\gamma = 6^\circ$ are more related to tooth root fractures, highlighting that the misalignment angle was the most detrimental variable.

Regarding conditions with misalignment, and considering the importance of the misalignment angle, there are analytical studies that have determined the relationship between the design parameters of the gear coupling and the maximum achievable misalignment angle [28], showing values very close to reality experimental analysis [4]. It has also been concluded that the number of teeth and the pressure angle are the predominant design parameters, followed by the crowning radius to define the maximum misalignment at which a crowned gear coupling can operate. Experimental studies, on the other hand, are more limited. For example, Herbstritt et al. [29] measured stresses at the tooth root with a maximum misalignment angle of $\gamma = 3^\circ$ and validated a finite element model with a 15% margin of error. However, there was limited information about the experimental methodology. More recently, Inurritegui et al. [4] presented a test rig where tests were conducted up to an angular misalignment of 10° . In this work, they also validated the number of teeth in contact, the contact pattern, and the stiffness of a coupling up to a misalignment of 7° with an error below 15%. However, there was no information regarding tooth root stress values in this case.

The literature has demonstrated that crowned gear couplings pose a series of challenges in terms of geometry and mechanical behaviour. Moreover, current standards still consider them as exceptional cases and do not provide precise sizing criteria. Recently, significant advancements have been made in the development of analytical and finite element models to determine the geometry of tooth surfaces and the mechanical behaviour of crowned gear couplings. However, a clear methodology for the design and sizing of couplings based on tooth root fracture criteria is still lacking. Therefore, the main objective of this study is to present the tools and methodology to be employed in designing and optimizing a crowned gear coupling that currently operates under high misalignment conditions, while also experimentally validating the stress distribution at the tooth root.

2 Methodology

In this section, a brief summary is provided of the analytical and numerical models that will be used for the design and sizing of gear couplings in this study. The case study employed is the gear coupling used in the rolling mill for high-strength steel. The high number of rolls in a reduced space, combined with the use of large gearboxes, results in a very high misalignment angle between the rolls and the gearbox, reaching angles greater than 6° [21,31]. The design conditions for this application are detailed in Table 1. It should be noted that the maximum applied torque corresponds to the central rolls operating under nearly aligned conditions. The applied torque on the outer rolls, that is, the most misaligned ones, decreases to approximately half (500 Nm).

Table 1. Design and operating conditions of the gear couplings in the rolling mill.

Parameter	Value	Unit
Maximum pitch diameter	50	mm
Maximum misalignment	6	°
Maximum torque	1000	Nm

The geometrical parameters of the gear couplings currently used in the rolling mill and which will be used for the experimental validation are listed in Table 2. The material from which they have been manufactured is a carburised and hardened 15NiCr11 steel with the mechanical properties set out in Table 2.

Table 2. Geometry parameters of the case study (Current case) employed on the rolling machine and material properties.

Parameter	Value	Unit
Pitch diameter	39	mm
Normal module	3	mm
Number of teeth	13	-
Pressure angle	30	°
Aspect ratio	0.30	-
Normal clearance	0.285	mm
Crowning radius	10	mm
Young modulus	210	GPa
Poisson coefficient	0.33	-
Density	7850	kg/m ³
Elastic modulus	850	MPa

2.1. Analytical geometry generation model

To analyse the mechanical behaviour and geometric characteristics of gear couplings, it is necessary to generate the geometry analytically. This involves considering both the manufacturing process and the geometry of the cutting tool. In the case of the hub, the hobbing process with a circular tool path and the generating gear hob, respectively, are taken into account. For the sleeve, the generating process using a shaper cutter and the shaper cutter itself are considered.

The spherical surfaces of the hub teeth are generated by considering the thread surface of the hob cutter as a set of cutting edges that act simultaneously during the

generation process, as explained in [3]. This is a two-parameter envelope generation process [30], where the thread surface of the hob cutter is determined prior to the generation of the hub.

First, the thread surface of the hob cutter (Σ_w) is generated as the cutting tool based on the rack cutter tooth surface (Σ_c). The hob cutter follows a circular feed motion that determines the shaft tooth surface (Σ_h) as the envelope to the family of hob cutter surfaces. The thread surface of the hob cutter is determined from the rack tooth surface Σ_c by considering the meshing equation (2). The hub tooth surface is generated by coordinate transformation (1) from the coordinate system S_c to the coordinate system S_h , taking into account the double enveloping process [30] with two independent generation parameters ϕ_w and s_w ((3), (4)).

$$r_h(u, v, \psi_w, s_w, \phi_w) = \mathbf{M}_{hw} \mathbf{M}_{wc} \mathbf{r}_c(u, v) \quad (1)$$

$$f_1(u, v, \psi_w) = \left(\frac{\partial \mathbf{r}_w}{\partial u} \times \frac{\partial \mathbf{r}_w}{\partial v} \right) \frac{\partial \mathbf{r}_w}{\partial \psi_w} \quad (2)$$

$$f_2(u, v, s_w, \phi_w) = \left(\frac{\partial \mathbf{r}_h}{\partial u} \times \frac{\partial \mathbf{r}_h}{\partial v} \right) \frac{\partial \mathbf{r}_h}{\partial \phi_w} \quad (3)$$

$$f_3(u, v, s_w, \phi_w) = \left(\frac{\partial \mathbf{r}_h}{\partial u} \times \frac{\partial \mathbf{r}_h}{\partial v} \right) \frac{\partial \mathbf{r}_h}{\partial s_w} \quad (4)$$

Here, ψ_w is the generation parameter for the thread surface of the hob cutter, and the matrices $\mathbf{M}_{hw}$ and $\mathbf{M}_{wc}$ are the coordinate transformation matrices between the coordinate systems. The matrices and model details can be found in [3].

For the sleeve of the coupling, the manufacturing process is shaper cutter generation, following the same steps defined for the hub. First, a hob cutter (Σ_s) is generated as a cutting tool based on the rack tooth surface (Σ_c) using the meshing equation (6). It follows a straight trajectory to determine the straight surfaces of the sleeve (Σ_g) as the envelope to the family of the shaper cutter surfaces. The coordinate transformation (5) from the coordinate system S_c to the coordinate system S_g , along with the consideration of the meshing equation (7) with the generation parameter ψ_s , allows to determine the sleeve tooth surface.

$$r_g(u, v, \psi_s, \phi_s) = \mathbf{M}_{gs}(\phi_s) \mathbf{M}_{sc}(\psi_s) \mathbf{r}_c(u, v) \quad (5)$$

$$f_4(u, v, \psi_s) = \left(\frac{\partial \mathbf{r}_s}{\partial u} \times \frac{\partial \mathbf{r}_s}{\partial v} \right) \frac{\partial \mathbf{r}_s}{\partial \psi_s} \quad (6)$$

$$f_5(u, v, \psi_s, \phi_s) = \left(\frac{\partial \mathbf{r}_g}{\partial u} \times \frac{\partial \mathbf{r}_g}{\partial v} \right) \frac{\partial \mathbf{r}_g}{\partial \phi_s} \quad (7)$$

Here, ϕ_s is the generation parameter for the sleeve surface, and the matrices $\mathbf{M}_{gs}$ and $\mathbf{M}_{sc}$ are the coordinate transformation matrices between the coordinate systems. The matrices and model details can be found in [3].

On the other hand, when experimentally validating the tooth root stress, the experimental geometry is measured using a coordinate measuring machine (CMM), from which the point cloud of the hub and sleeve tooth surfaces is extracted. The reconstruction of the surfaces is carried out using the methodology described in [4], which eliminates unnecessary points and generates an ordered point mesh suitable for the three-dimensional meshing required for subsequent finite element simulation.

2.2 Finite element model

To analyse the mechanical behaviour and contact trajectory of crowned gear couplings, the finite element method is employed. For this purpose, the method presented by Argyris et al. [31], commonly used for gear analysis, is employed to mesh the generated toothed surfaces. This well-known method produces a structured mesh on the gear teeth, dividing the surface into five sections: active profile, tooth root on the right and left sides, and the gear body. By analytically generating the tooth geometry using the model presented in the preceding section, the nodes forming the tooth surface are known. From these, the hexahedral element mesh is automatically generated inward for the hub and outward for the sleeve. Figure 1(a) shows a detail of the mesh used on the hub as an example. The element size is reduced in the potential contact region of the tooth, providing smaller elements in areas with higher stress gradients without increasing the number of elements and, therefore, the computational cost. As seen in Figure 1(a), the fine mesh zone corresponds to the region where contact will occur, which is defined by a contact analysis (TCA) [28], and an additional 25%. The element size gradually increases toward the edges of the coupling. Several layers of small elements ($l_{\min} = 0.08$ mm) characterized by thickness $e = 0.15$ mm [32] are generated on the tooth surface, which transition to larger elements in the body of the hub or sleeve.

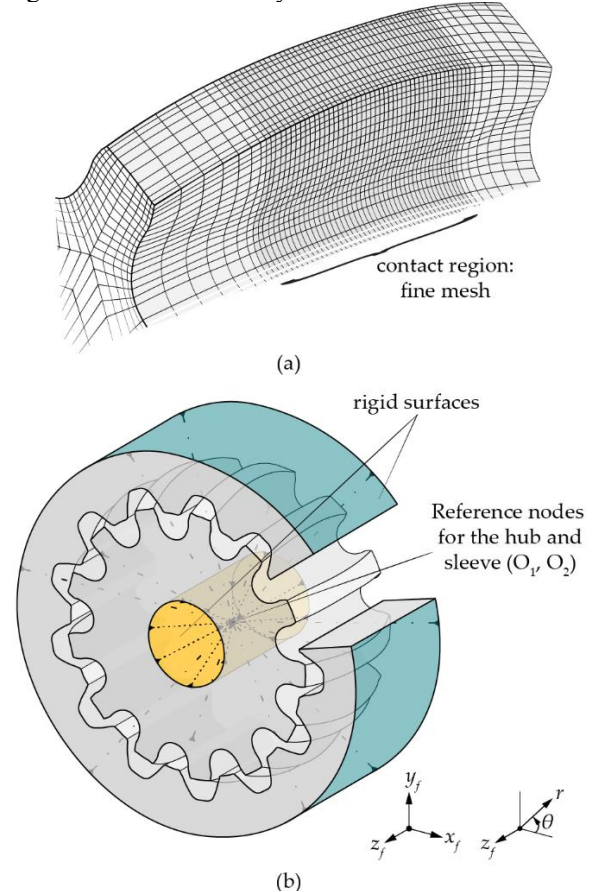


Fig. 1. (a) discretization of the longitudinal meshing of the finite element model with the identified contact zone, and (b)

description of the boundary conditions and rigid surfaces defined in the finite element model (adapted from [17]).

The shaft and sleeve are assembled in a fixed coordinate system S_r . The auxiliary coordinate system S_m allows applying the misalignment angle by rotating the sleeve around the x -axis while keeping the hub fixed during the analysis. The origins of both coordinate systems are located at the central section of the coupling (0,0,0), where the reference nodes of the shaft and sleeve are also placed. These nodes are rigidly connected to the nodes located on the inner diameter of the hub and the outer diameter of the sleeve, respectively, to form rigid surfaces (shown in yellow and green in Figure 2(b)). Additionally, to perform the simulation and ensure proper post-processing of the data, a cylindrical coordinate system (r, θ, z_f) [17] is defined. During the simulation, the degrees of freedom of the sleeve are blocked, while in the hub, all degrees of freedom are blocked except for the rotation around the z -axis, allowing for the application of torque.

In these simulations, friction between the components is not taken into account, and contact is defined using node-to-segment contact pairs, with a tolerance equivalent to $1/20^{\text{th}}$ of the smallest element size. Additionally, a linear elastic material is assumed (properties listed in Table 2), under the assumption that the deformations are small enough to be studied using the theory of small deformations. The selected element type is a first-order isoparametric hexahedral element (type 7 in the Marc simulation software [33]).

2.3 Test rig and experimental setup

For the experimental validation, [4] presents a specially designed static test rig to test crowned gear couplings with high misalignment angle ($\gamma_{\max} = 10^\circ$), which was used for this study. The test rig can be seen in Figure 2(a) and consists of two main parts: the static part, where the sleeve is placed and the torque is applied, and the movable part, where the hub is placed and the angular misalignment is applied in the clockwise direction. This is achieved using a pivoting system with a series of holes in the movable part and the base of the test rig, along with positioning pins, which allow for adjusting the angular misalignment degree by degree with an accuracy of $\pm 0.2^\circ$. The torque is applied using calibrated weights up to a maximum of 1000 Nm and is measured using a full-bridge torque strain gauge (CEA-06-250US-350 by Micromeritics), which was calibrated with a high-precision torque measurement unit to achieve an accuracy of ± 20 Nm.

The most commonly used method in the literature for empirical evaluation of the tooth root stresses is to use strain gauges [24,30,36]. Focusing on studies conducted on gear couplings, [34] numerically and experimentally analysed the load distribution along the length of the coupling, and [24] instrumented a tooth across the entire face width with miniature strain gauges. Quarter-bridge strain gauges and a data acquisition frequency of 100 Hz were used.

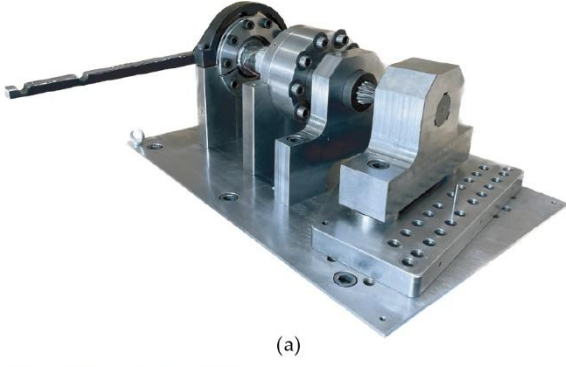
Therefore, in order to experimentally validate the stresses in the tooth root, based on the geometry of the coupling (properties in Table 2), all teeth except one will be machined to allow for proper attachment of the strain gauges and eliminate any convective effects from adjacent teeth. A quarter-bridge strain gauge (1-LY11-0.6/120 by HBM) with a wire width of 1 mm is placed on both sides of the tooth, as shown in Figure 2(b). These strain gauges allow for localized uniaxial strain measurements and can measure tension and compression caused by tooth bending.

The test conditions for the experiments are from 0 Nm to 150 Nm (since there is only a single tooth bearing the entire applied torque) and from 0° to 7° of angular misalignment, with five repetitions per test to ensure result repeatability and eliminate any clearance in the mechanism. Once the torque is applied using the calibrated weights, data acquisition is performed for 30 seconds to extract the average strain value for each strain gauge.

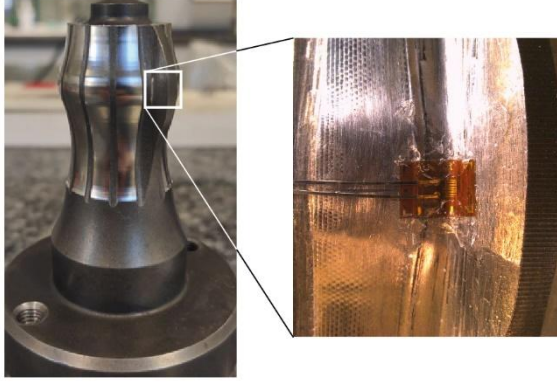
3 Results and Discussion

3.1 Experimental validation

In Figure 3, the results obtained from both the numerical model and the experimental tests are shown for different misalignment angles and as a function of the applied torque. The error bars displayed in each experimental data point represent the variability of the results obtained in all the repetitions conducted. Additionally, the shaded area represents the numerical values obtained within the torque precision range of $T \pm 20$ Nm, corresponding to the location of the strain gauge. The experimentally measured deformation values have been converted to stresses using



(a)



(b)

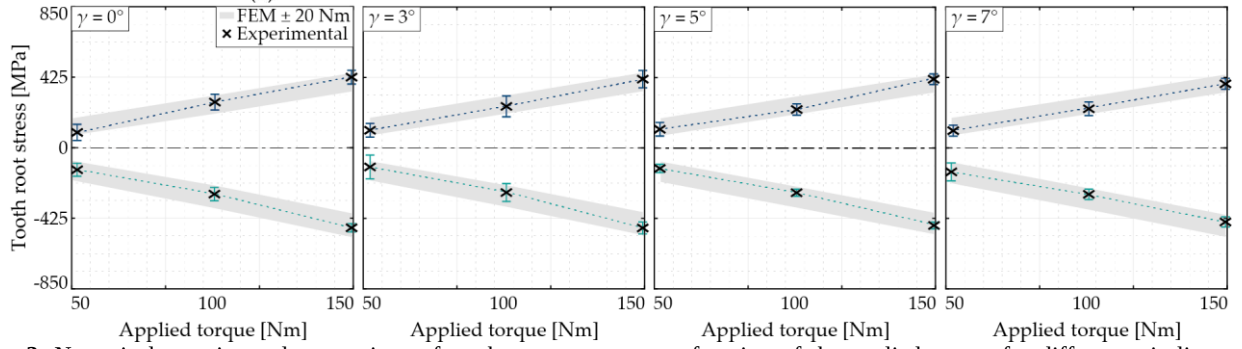


Fig. 3. Numerical-experimental comparison of tooth root stresses as a function of the applied torque for different misalignment angles.

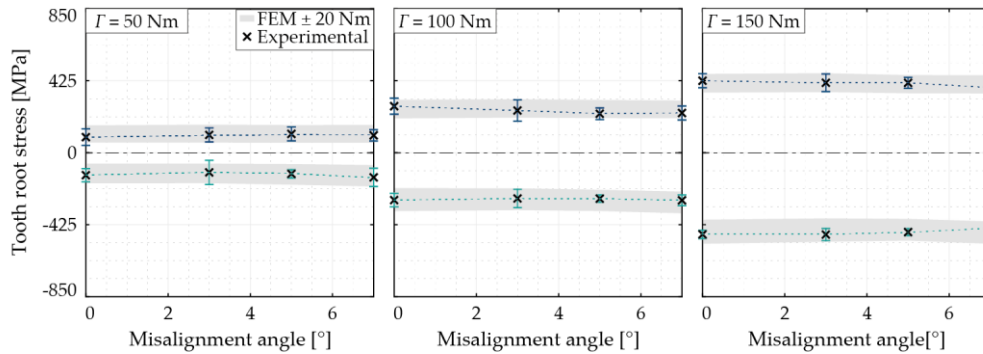


Fig. 4. Numerical-experimental comparison of tooth root stresses as a function of the misalignment angle for different applied torques.

the modulus of elasticity (Table 2): $\sigma = E\varepsilon$, to make the data more representative. As observed in the graphs, higher applied torques result in higher stresses at the tooth root. Additionally, compressive stresses are slightly higher than tensile stresses, consistent with the literature [35]. Furthermore, the experimentally obtained values fall within the range of the calculated numerical values. It should be noted that the observed differences

could be attributed to variations while putting the gauges or the direction not being complete perpendicular to the tooth.

Furthermore, Figure 4 shows the evolution of tooth root stress in both the tension and compression sides, as a function of the misalignment angle for different applied torque values. The trend is similar for all torques, with the differences due to the influence of the misalignment

angle becoming more significant at higher applied torques. The tooth root stress value does not vary significantly with the misalignment angle. This is because the stress is measured in a single tooth and single face width position. Therefore, the tooth root stress variation is only affected by the modification of the tooth stiffness caused by the displacement of the contact point.

This effect is better understood in Figure 5, which shows the normalized numerical tooth root stress values for different misalignment angles. The peak stress value shifts along with the contact point (Figure 5(a)), and the variation in the stress value does not exceed 10% (Figure 6(b-c)). Similarly, when looking at the stresses at the same point ($b = 0$ mm) for different misalignment angles (as done experimentally), the variation in stress is less than 20%. Experimentally, the differences are less pronounced due to the precision in the applied torque, the proximity between the contact points, and the error in the positioning of the strain gauges.

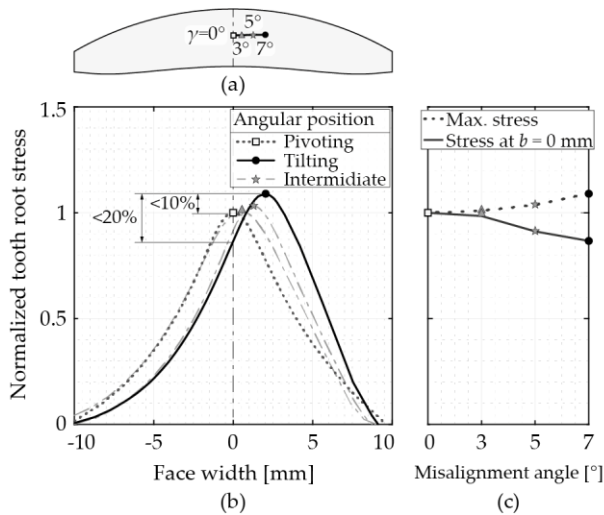


Fig. 5. Numerical results: (a) evolution of the contact point and (b) normalized tooth root stress as a function of the misalignment angle. (c) Variation of the normalized maximum tooth root stress as a function of the misalignment angle.

3.2 Methodology for the optimization of crowned gear coupling

As mentioned in Section 2, the described gear coupling in Table 2 is currently used in a high-strength steel rolling mill. Continuing with the coupling requirements specified in Table 1, this subsection identifies the optimal coupling geometry for this application using the models described in Sections 2.1-2.2.

Firstly, using the pre-design charts provided in [28], the parameters of the gear coupling geometry that can achieve a maximum angular misalignment of 6° are chosen. These charts are selected according to the desired pitch diameter, in this case, 50 mm since it is the maximum allowed pitch diameter, and a smaller pitch radius would have a higher risk of tooth interference for high angular misalignments [28]. The charts also indicate the limits of undercut and pointed teeth, which

should be avoided in a redesign as they weaken the life of the coupling. It is deduced that to achieve interference-free geometries that reach the desired misalignment, it is necessary to choose geometries with a reduced number of teeth or larger modules ($m = 5 - 6$ mm). Additionally, higher pressure angle allows achieving the desired angular misalignments without tooth interference or undercut. Based on these pre-design charts, the $\alpha = 20^\circ$ geometries are discarded since tooth interference during the manufacturing process is inevitable to achieve the desired angular misalignment. Therefore, three geometries with $d_p = 50$ mm are selected, and three additional geometries with $d_p = 40$ mm (which matches the current coupling) to compare and decide which is the optimal geometry. The properties of the preselected geometries for the analysis are described in Table 3.

Table 3. Parameters of the preselected case studies numerically analysed.

	C 1	C 2	C 3	C 4	C 5	C 6
d_p , [mm]	50	50	50	40	40	40
m_n , [mm]	6	6	6	5	5	5
z , [-]	8	8	8	8	8	8
α , [-]	30	37.5	45	30	37.5	45
b/d_p , [-]	0.3	0.3	0.3	0.3	0.3	0.3
r_c , [mm]	24	27	27	20	20	23
Undercut	No	No	No	No	No	No
γ_{max} , [°]	6.7	7.2	8.5	6.6	8.2	8.3

After performing the simulation with the maximum misalignment angle ($\gamma = 6^\circ$), the two parameters that influence the maximum stresses in the tooth are analyzed: the number of teeth in contact and the effective face width. Figure 6 shows the normalized number of teeth in contact and the normalized effective number of teeth in contact for each of the preselected cases. By solely considering the number of teeth in contact (Figure 6(a)), it is concluded that Case 4 is not suitable since it has the lowest number of teeth in contact for maximum misalignment, while Case 1 is the most appropriate. In other words, a larger pitch diameter with the same geometric properties of the teeth (except for the module in this case) provides a greater number of teeth in contact, resulting in lower tooth root stresses. However, previous studies [12,15,17,39] have shown that, due to the non-uniform load distribution in the presence of misalignment, the effective number of teeth in contact (Figure 6(b)) is a more relevant parameter for calculating the stresses in the tooth. In this comparison, it can be seen that there is not much variation in the results obtained among the analyzed cases. That is, all the chosen cases exhibit a similar load distribution, divided among approximately the same percentage of the tooth face width. This may be due to the low number of teeth in these geometries. Additionally, given the high degree of angular misalignment, even with an increase in

the applied torque, the number of teeth in

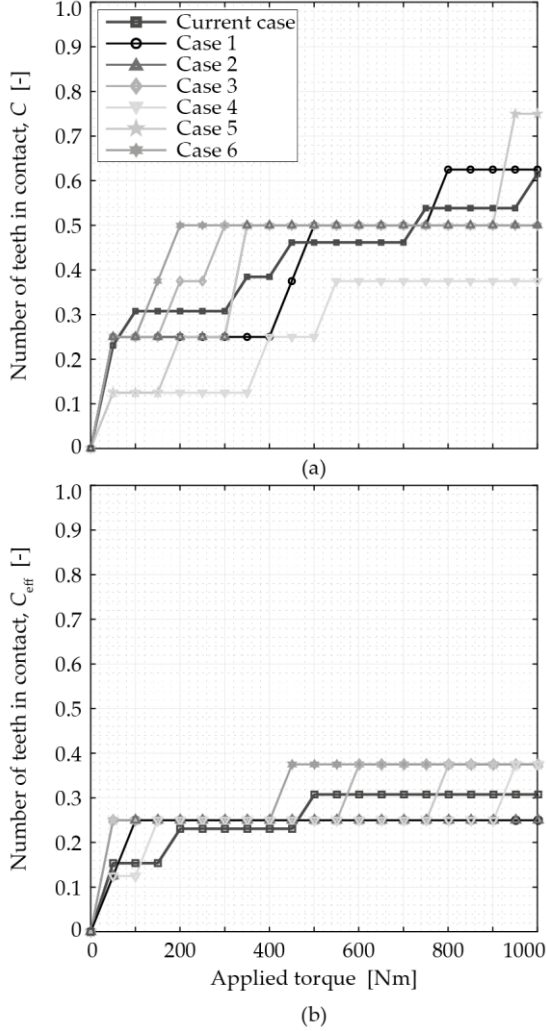


Fig. 6. (a) number of teeth in contact and (b) number of teeth in effective contact normalized for the preselected cases as a function of applied torque.

contact does not increase.

On the other hand, Figure 7 shows the effective face width calculated from the simulation results. It can be observed that, for the same aspect ratio, the effective face width is greater in cases with a smaller pitch diameter ($d_p = 40$ mm). This means that having a larger face width does not necessarily mean that this width is bearing the load. Cases 4, 5, and 6 have an effective face width of approximately 75% of the total length of the coupling, while cases 1, 2, 3, and the current case have a 50% effective face width. This indicates that cases 4, 5, and 6 are more optimized compared to the current case, as they have a larger tooth width supporting the load. No effect of the pressure angle on the effective face width is observed. In Figure 8, the stress distribution in the tooth for the maximum applied torque ($T = 500$ Nm) with an angular misalignment of $\gamma = 6^\circ$ can be seen. There is a difference between the current case and the rest of the generated cases in terms of the position of the maximum stress value position (corresponding to the contact point). The parameter that influences this is the crowning radius, which is 10 mm in the current case and between

20-27 mm for the other generated cases. A smaller crowning

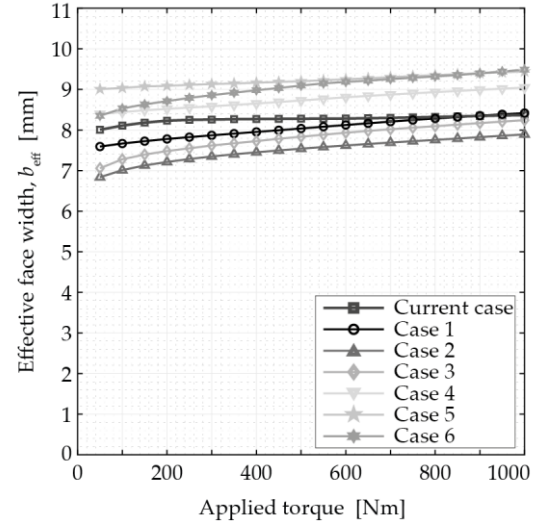


Fig. 7. Effective face width as a function of applied torque.

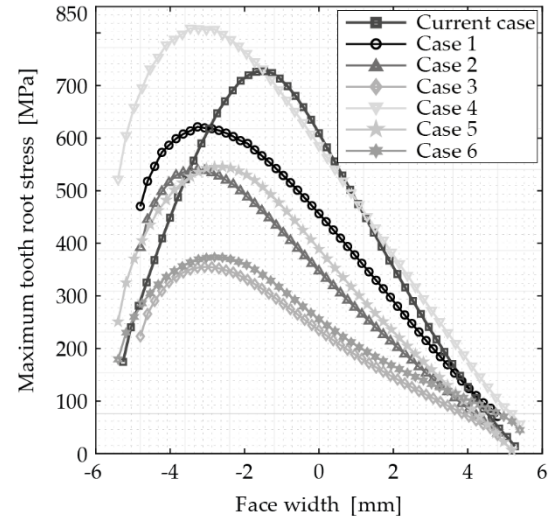


Fig. 8. Maximum stress in the tooth along the face width for the preselected cases and an angular misalignment of $\gamma = 6^\circ$.

radius for the same angular misalignment results in the contact being more centred on the tooth width. However, reducing the crowning radius increases the risk of interference during manufacturing. It can be observed that the influence of the pitch diameter on the stresses is less pronounced with higher pressure angles. Lower stresses are obtained with $\alpha = 45^\circ$ due to the greater tooth thickness.

After analyzing the two main parameters influencing the stress in the tooth, Figure 9 shows the evolution of the maximum tooth root stress as a function of the applied torque. A higher pressure angle results in a reduction of stress of over 50% compared to the current case. This reduction is primarily due to the increased tooth thickness provided by the coupling manufactured with $\alpha = 45^\circ$, as the influence on the number of teeth in contact is not evident. Additionally, it has been observed that a larger primitive diameter does not offer

remarkable advantages in the analyzed case. Indeed, the stresses are higher despite having a wider face width to bear them, in cases with a smaller pitch diameter.

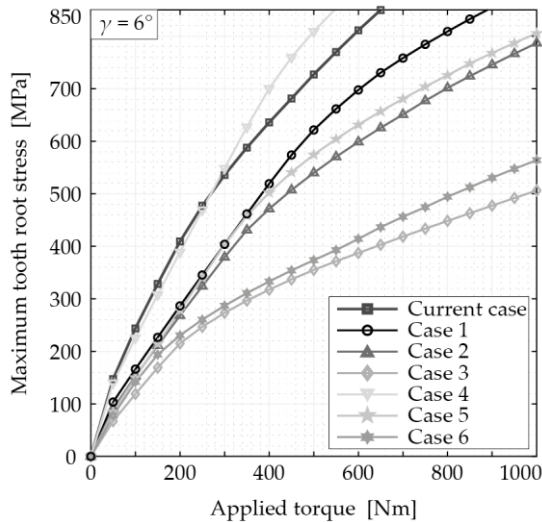


Fig. 9. Maximum stress in the tooth for the preselected cases as a function of applied torque for $\gamma = 6^\circ$.

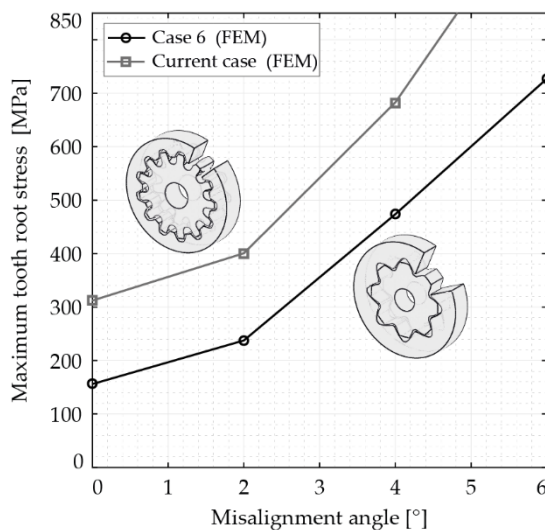


Fig. 10. Numerical-analytical maximum stress in the tooth for the Current Case and the optimized case (Case 6) as a function of the misalignment angle $T = 1000$ Nm.

Therefore, to conclude that case 6 is suitable for improving the current employed geometry, Figure 10 shows the evolution of the maximum tooth root stress as a function of the misalignment angle for the maximum applied torque. A reduction of approximately 50% in stress is confirmed across the entire range of misalignment angles. Thus, with the optimized design, it would be possible to increase the torque applied to the rollers or increase the lifespan of the couplings, even at the highest misalignment angle.

4 Conclusions

This work shows a compilation of analytical and numerical models for the determination and calculation of tooth root stresses in crowned gear couplings working in high misalignment applications. It experimentally validates the numerically obtained stresses and proposes a methodology to optimize existing designs or even to realize new designs. The results obtained show the effectiveness of the numerical models, proposing a design with which it is possible to reduce the tooth root stresses by more than 50%. This change will extend the useful life of the component, as well as increase the maximum applicable torque. All of these are advantages for the user, manufacturer or seller of the rolling machine.

It has been shown that the effective number of teeth in contact is a key parameter to determine the tooth root stress and that it has little to do with the number of teeth in contact. Furthermore, it is concluded that the pressure angle has a great influence on the reduction of tooth root stresses in gear couplings working in high misalignment angles. Likewise, it has been found that the effective face width does not vary excessively among the cases analysed herein.

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Improving the wear resistance of 3D printed spur gears through a free-form tooth flank optimization process

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Abstract. Involute gears have traditionally been the preferred choice for gear transmission systems due to their simplicity and interchangeability. However, there are applications where they do not provide the most durable and efficient solution. While the cost of implementing optimized non-involute gears in most applications often outweighs their comparative advantages, the advent of additive manufacturing has opened up possibilities for designers to explore alternative gear tooth profiles. This is particularly relevant in the realm of plastic gears, where optimized non-involute gears produced through 3D printing can address their primary drawbacks, such as surface durability and wear resistance. In this study, a comprehensive free-form optimization process was conducted to determine the optimal tooth profile that minimizes wear on 3D printed spur gears during operation. The tooth flank geometry was represented using a 4th order B-spline curve, and a genetic algorithm was employed to determine the optimum positions of the control points aiming to minimize wear depth across the tooth flanks. The spur gears were manufactured using Fused Deposition Modeling (FDM) with PLA material. The parameters of the additive manufacturing process were experimentally fine-tuned to achieve the best possible accuracy. To evaluate the performance of the optimized free-form gears, two case studies were implemented, demonstrating that the optimized gears achieved a remarkable reduction of average wear depth by more than 50% and a reduction of maximum wear depth by more than 69% compared to standard involute gears. To further validate the effectiveness of the optimization method, experiments were carried out using an FZG test rig. The profiles of the tooth flanks were measured on a Coordinate Measuring Machine (CMM) before and after the experiments to compare the wear depth against the standard involute gears. The results revealed a significant improvement in the wear resistance of the tooth flanks, with a reduction of wear depth of 44.1%.

1 Introduction

Gears are widely regarded as the most commonly utilized mechanical transmission system across various industries, including automotive, aerospace, agriculture, mining, and energy [1]. Despite their diverse applications with distinct requirements, the vast majority of gear transmissions feature the same tooth flank profile, which follows the involute curve. Involute gears offer several advantages, such as insensitivity to variations in center distance, simplicity and robustness in manufacturing [2, 3, 4, 5]. However, depending on the specific application, they also possess certain drawbacks such as limited load capacity due to their relatively poor pitting resistance, suboptimal lubrication characteristics, and proneness to interference [2, 6, 7].

To address these drawbacks, researchers and gear designers have proposed alternative gear profiles that

can enhance the performance of gear transmissions by increasing efficiency and load capacity or reducing noise and vibrations. Among the well-known alternatives are cycloidal or circular-arc gears (i.e., Novikov gears) that offer improved efficiency and load capacity, respectively. However, their performance in practical applications is often worse than that of involute gears due to their high sensitivity to manufacturing and assembly errors [2].

In addition to cycloidal and circular arc gears, researchers have introduced a wide variety of tooth profiles. Komori et al. [6] proposed the use of a rack-cutter profile composed of many continuously divided minute involute curves to create the "LogiX" tooth profile, which offers twice the durability of involute gears. Tsai & Tsai [8] developed a method for designing high-contact-ratio gears based on a quadratic function of the pressure angle to improve load capacity and NVH

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characteristics. Litvin et al. [9] implemented a parabolic rack-cutter profile to produce a modified Novikov-Wildhaber double-crowned helical gear drive, resulting in reduced noise and vibrations and enhanced load capacity. Zhang-Hua et al. [10] proposed a novel tooth profile based on a hybrid line of action, which incorporates straight line and circular-arc segments to control curvature, sliding velocities, and contact ratio of the meshing gears. Luo et al. [4] introduced a cosine gear profile that exhibits lower sliding coefficients and reduced contact and bending stresses compared to standard involute gears. Wang et al. [11] developed a tooth flank profile based on a parabolic line of action, aiming to reduce contact and bending stresses and improve efficiency.

All of the aforementioned approaches aim to enhance the performance of gear pairs by initially introducing a predetermined tooth flank profile and subsequently evaluating its performance. However, it would be advantageous for gear designers to have direct control over the optimization of specific performance objectives. In this regard, Liu et al. [12, 13] developed tooth profiles based on a predefined distribution of relative curvature, resulting in improved bearing capacity and lubrication characteristics. Additionally, Wang et al. [14] proposed a gear design method that focuses on achieving higher efficiency by utilizing given sliding coefficients.

Taking a further step, Yeh et al. [2] proposed an optimization methodology that allows for control over the shape of the tooth flank to improve multiple performance objectives, including load capacity and efficiency. This approach utilizes a deviation function technique based on Non-Uniform Rational B-Splines (NURBS) to control the shape of the tooth profile. Yu & Ting [5] introduced a free-form conjugation modeling technique that enables the modification of the rack-cutter profile or the contact path to achieve desired performance characteristics. B-spline curves are employed to model these profiles, and certain modeling challenges are discussed, such as undercutting or cusps in the conjugate profiles. Furthermore, Kalligeros et al. [15] presented a comprehensive free-form optimization method aimed at enhancing the pitting resistance of spur gear tooth flanks. The tooth profiles are represented as B-spline curves, and the positions of their control points are optimized to achieve a minimum and uniform distribution of relative curvature along the contact path.

Despite these advancements in alternative gear designs, the industry has shown reluctance to incorporate them into their applications. This reluctance primarily is derived from the substantial effort and cost associated with changing and fine-tuning the manufacturing processes, which are predominantly tailored for involute gears. These obstacles often outweigh the potential benefits gained from utilizing non-involute gear designs [5, 16]. However, the emergence of additive manufacturing and 3D printing as viable alternative manufacturing processes can address these challenges. Unlike traditional manufacturing methods, which often require complex machining processes, 3D printing enables the production of gears with intricate, free-form geometries that were previously difficult to achieve [17].

This capability allows engineers and designers to optimize gear designs for specific applications, resulting in improved performance, reduced weight, and enhanced functionality [18]. Versatility and design freedom provided by 3D printing technology are predominant advantages of plastic gears, along with lightweight construction, self-lubrication and reduced manufacturing costs [19].

However, alongside the advantages of plastic 3D printed gears, wear emerges as a major concern [20]. Wear is the gradual loss of material that occurs when two surfaces slide or roll against each other and it can have a profound impact on the performance and longevity of plastic gears. In the context of 3D printed plastic gears, the unique material properties, manufacturing processes, and gear geometry all play significant roles in the resulting wear.

In an attempt to minimize wear, Zhang Ye et al. optimized 3D printing parameters (printing speed and temperature, infill, and bed temperature) using ANNs, achieving a remarkable 300% reduction in wear [21]. Tunalioglu et al. introduced models to predict the wear of FDM plastic gears [22]. However, determining the wear coefficient for plastic gears in the case of 3D printing remains an open issue, as standard tests such as pin-on-disc tests are not adequate [23].

This study focuses on the geometrical optimization of gear tooth flanks to maximize their wear resistance. A comprehensive optimization algorithm is developed, treating the tooth profiles as B-spline curves and determining the optimal placement of their control points through a genetic algorithm to produce flanks with desirable wear characteristics. The optimized free-form curves achieved a remarkable reduction of wear by over 50%. The results were validated through experiments on both standard involute and free-form gears using an FZG testing machine and a CMM measuring machine. The experimental results revealed that the optimized gears experienced 44.1% less wear than their involute counterparts.

2 Free-form optimization algorithm

Building upon the research presented in [15], the objective of the optimization process is to establish a systematic algorithmic procedure capable of generating optimized tooth flanks that adhere to the fundamental principles of gearing. These optimized profiles should ensure the absence of undercutting, interference, cusps, or any other geometric or operational issues that may arise.

The optimization process is structured as follows:

1. The tooth flank profile is represented by a B-spline curve with a specified number of control points and polynomial curve order.
2. The positions of the control points are adjusted using a genetic algorithm.
3. The resulting profile is evaluated to determine if it satisfies the law of gearing at each point.

4. If the profile meets the requirements, the conjugate profiles (i.e., the conjugate gear profile, rack-cutter profile and contact path) are calculated implementing the established analytical theory of gearing and the involutization method.
5. Based on these profiles, the objective function is calculated.
6. In case the produced profiles violate any constraints related to their geometric or operational characteristics, a penalty is applied to the objective function.
7. The entire procedure is iteratively repeated until convergence is achieved.

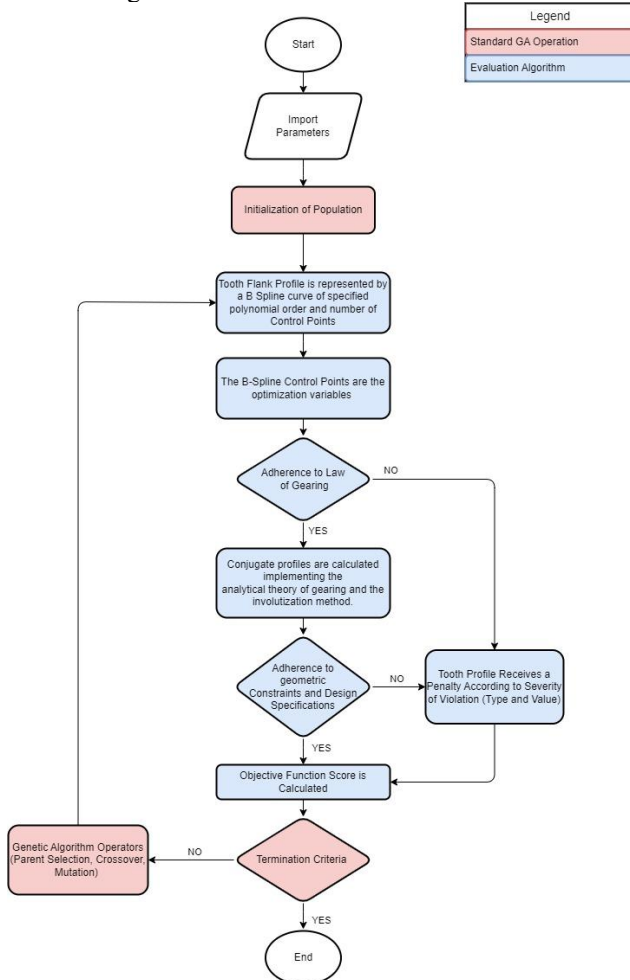


Fig. 1. Flow chart of free-form tooth flank optimization algorithm.

2.1 Modeling of tooth profile

In non-involute gear design, it is often preferred to optimize the rack cutter or the contact path profile to indirectly enhance the performance of the conjugate tooth flanks [5]. While this approach can be meaningful in certain cases (i.e., to ensure an easy-to-manufacture rack cutter profile), there is a desire among designers to directly control the shape of the tooth flank, which in turn directly affects the desired performance improvements. However, a common challenge arises when applying Buckingham's analytical theory of gearing [24], as it requires solving an implicit equation

to calculate the conjugate profiles [15]. Incorporating a numerical convergence algorithm to solve this equation would significantly increase computational cost and potentially undermine the reliability and robustness of the optimization process. This is one of the reasons why gear designers tend to avoid utilizing the tooth flank as the primary geometry for optimization.

This limitation can be overcome by employing the involutization method introduced by Spitas et al. [25, 26]. In this method, every point of the tooth profile is associated with a local involute curve. By applying the fundamental principles of gearing and utilizing the inherent properties of involute curves, it becomes possible to explicitly compute the contact path of the gear pair. Once the contact path is determined, all the corresponding conjugate profiles can be derived utilizing the analytical theory of gearing [24].

More specifically, when considering a random point G on the tooth flank characterized by $y = f(x)$ that has an inclination of $df/dx|_G$, it is possible to associate a local involute curve with G . This local involute curve is defined by a local base circle and a local pressure angle that intersects with G and shares locally the same inclination. The calculations for the local base circle and pressure angle are as follows [25]:

$$r_{gG} = \frac{x_G - f(x_G)df/dx|_{(x_G, y_G)}}{\sqrt{1 + (df/dx)^2|_{(x_G, y_G)}}} \quad (1)$$

$$a_{oG} = \cos^{-1}(r_{gG}/r_0) \quad (2)$$

where r_{gG} and a_{oG} the local base circle and pressure angle respectively, $x_G, f(x_G)$ the coordinates of G and r_0 the pitch circle of the pinion.

Based on the local involute curve, the coordinates of the contact point that corresponds to G can be determined as:

$$x_P = r_{gG} \left[\sqrt{\frac{r_G^2 - r_{gG}^2}{r_0^2}} - \sqrt{1 - \left(\frac{r_{gG}}{r_0}\right)^2} \right] \quad (3)$$

$$y_P = x_P \sqrt{\left(\frac{r_0}{r_{gG}}\right)^2 - 1}$$

where x_P, y_P the coordinates of the contact point and r_G the radius of G .

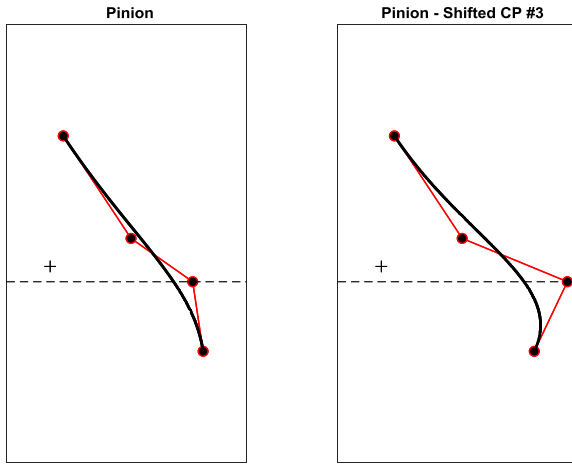
By applying this process to each point along the tooth flank, the complete contact path can be generated. Subsequently, the conjugate gear profile can be explicitly calculated using the analytical theory of gearing, as outlined below:

$$\begin{aligned} x_2 &= x_p \cos \theta_2 - (y_p - r_{o2}) \sin \theta_2 \\ y_2 &= x_p \sin \theta_2 + (y_p - r_{o2}) \cos \theta_2 + r_{o2} \\ r_{o2} \theta_2 &= -r_o \theta \\ \theta &= \frac{1}{r_o} \int_{x_0}^{x_p} \left(1 + \frac{y_p}{x_p} \frac{dy_p}{dx_p} \right) dx \end{aligned} \quad (4)$$

where x_2, y_2 the coordinates of the conjugate gear profile, r_o and r_{o2} the pitch circles of the pinion and the conjugate respectively, θ and θ_2 the rotation angles of conjugates points of the pinion and the conjugate gear respectively in order to mesh.

To establish a truly free-form tooth flank optimization process, it is necessary to find an efficient and convenient method for modifying the tooth profile. One highly effective approach is to represent the tooth flank as a parametric curve, such as a B-spline or Bezier curve, which can be easily adjusted by manipulating the positions of a small number of control points. These control points serve as the variables in the optimization problem, with the objective of the optimization being to determine their optimal placement.

For the optimization problem at hand, B-spline curves are employed due to their superior local control compared to other parametric curves (i.e., Bezier curves). Fourth-order polynomials are utilized whenever possible, as higher-order B-spline curves yield smoother curves – a desirable characteristic for tooth flanks [2]. The determination of the appropriate number of control points is specific to each case and their position should be subject to certain constraints, which will be examined in the subsequent section. As illustrated in Fig. 2 only the portion of the tooth flank that engages with the conjugate gear profile is modeled since it is the section of interest for this optimization problem. The lower part of the tooth is generated once the optimization process is finalized. In Fig. 2, it is also demonstrated how changing the position of the control points can affect the tooth profile. Even an adjustment to just one control point can result in a significant alteration in the shape of the tooth



profile.

Fig. 2. Influence of the position of the control points of the B-spline curve to the tooth profile.

2.2 Optimization constraints

The gear design community has faced a challenge in establishing a well-defined set of constraints to effectively assess the suitability of the generated free-form tooth flanks [5]. Firstly, the produced tooth flanks should be free from issues like undercutting or interference during operation. Additionally, the generated tooth profiles should not result in conjugate gear geometries that exhibit cusps or other discontinuities. At the same time, it is essential to ensure that the contact ratio remains within an acceptable range and that the profiles adhere to the law of gearing at each point.

To prevent interference during operation, it is crucial to ensure that the B-spline curve representing the tooth flank includes the lowest point of tooth contact of the pinion (i.e., the generating curve). This enables the creation of a corresponding conjugate point on the gear profile, thus avoiding interference. Since the radius of the lowest point of tooth contact varies for each case, the control point at the lower end of the tooth profile should be positioned to accommodate all possible scenarios. If n is the number of control points and the n th control point is located at the lower point of the tooth flank, the first constraint for its radius can be expressed using the following relation.

$$r_{cp,n} = a_{12} - r_{k,2} \quad (5)$$

where $r_{cp,n}$ is the radius of the n th control point, a_{12} is the center distance and $r_{k,2}$ is the addendum circle of the conjugate gear. Given the addendum circle of the gear, no contact can occur between the meshing gears below this radius ($r_{cp,n}$).

To prevent undercutting, two measures are implemented. Firstly, the addendum and dedendum coefficients of both gears can be adjusted to address potential undercutting issues. If these adjustments are insufficient, an upper limit should be imposed on the x-coordinate of the n th control point. The precise value for this limit can be determined after a few optimization iterations. Furthermore, setting an upper bound on the x-coordinate of the n th control point helps prevent the generation of a "pointy" tooth (i.e., a tooth with an excessively narrow top land width) in the conjugate gear.

To address the same issue in the pinion, it is necessary to establish a lower bound for the x-coordinate of the first control point located at the tip of the tooth flank. According to [27], the narrowest width of a tooth top land should be 20% of the gear module. Consequently, if the y-axis of the absolute coordinate system aligns with the tooth's axis of symmetry, the lower bound should be set at $0.1m$, where m is the module of the gear pair.

For the remaining control points, while specific bounds may not be necessary, leaving them entirely unconstrained can result in convergence issues. This is because in such a case the majority of generated profiles tend to not adhere to the optimization constraints. After a large number of optimization iterations, a robust approach to setting bounds for the control points is as follows: an involute curve is initially generated based on the gear parameters, which is then interpolated using a B-spline curve with the desired number of control points. The bounds for the control points are then defined as a range around these values. This procedure ensures that a sufficient number of the generated curves comply with the constraints, allowing the optimization algorithm to converge. However, it is essential to ensure that the bounds are large enough to avoid guiding the optimization towards involute-like curves. Alternatively, different arbitrary bounds can be implemented, which it was proved that they lead to similar results after sufficient iterations of the optimization algorithm.

Another crucial constraint that needs to be considered is the contact ratio ϵ of the free-form gears. For a spur gear pair, the contact ratio should exceed 1, with ISO and

AGMA standards recommending values above 1.2 or 1.3 to ensure smooth operation [28, 29]. However, achieving a sufficient contact ratio for optimized free-form gears has proved to be a challenge, as it is typically a contradicting parameter with efficiency. To ensure an appropriate contact ratio for the optimized gears, a penalty is incorporated into the objective function. This penalty becomes higher for lower contact ratio values. After experimenting with various expressions, the following one was selected that has the distribution illustrated in Fig. 3 and is added as a factor in the objective function.

$$W_{\varepsilon} = (10 * e^{-\varepsilon} - 10 * e^{-1.5})^4 + 1 \quad (6)$$

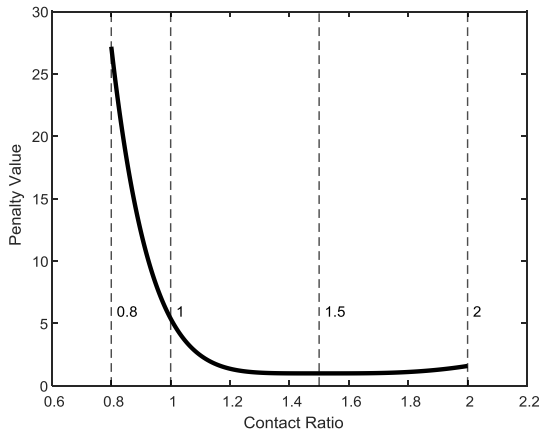


Fig. 3. Distribution of penalty value in relation to contact ratio.

In addition to the aforementioned well-known constraints that apply to standard involute gears, when dealing with free-form gears it is necessary to establish additional constraints to ensure that the generated profiles adhere to the law of gearing and are manufacturable, without any cusps or other discontinuities. After generating a large number of free-form profiles, two major problems have emerged during the calculation of the conjugate geometries. Firstly, some points encounter calculation issues as they lead to negative radicands. Secondly, conjugate profiles with cusps and C1 discontinuities are frequently generated, as depicted in Fig. 4.

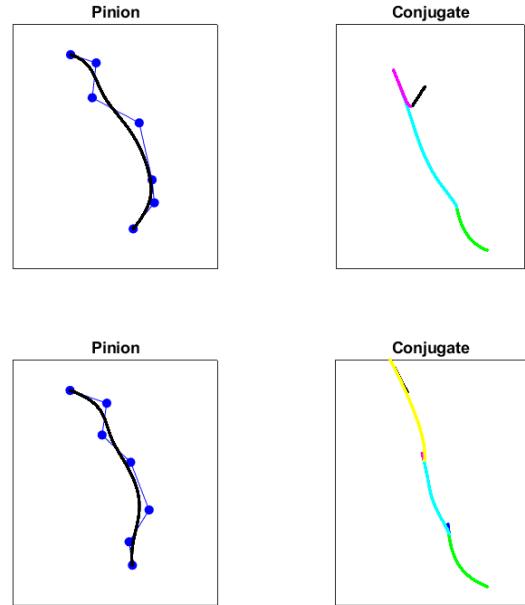


Fig. 4. Cusps and C1 discontinuities of conjugate profiles (right) generated from free-form profiles (left).

When the calculation is impossible at certain points, it indicates that these points cannot satisfy the law of gearing. In other words, regardless of the point's rotation, its normal vector will never intersect the pitch point of the pinion. Free-form profiles that contain such points cannot function as tooth flanks and should be heavily penalized by the optimization process.

To address the problem of discontinuities in the calculation of conjugate profiles, certain measures are implemented. As depicted in Fig. 4 when such issues arise, the resulting conjugate profile appears to consist of multiple distinct segments. For each segment, an additional penalty is applied as a factor to the objective function, since a high number of these segments indicates that the free-form profile deviates significantly from being a valid tooth flank.

2.3 Wear model

As previously stated, the primary objective of the optimization process is to propose alternative tooth flank profiles that exhibit improved wear resistance during operation. To establish a comprehensive objective function, the wear model presented in [30, 31] is incorporated.

The wear model is based on the well-known Archard's wear equation that is typically expressed as:

$$\frac{V}{s} = K \frac{W}{H} \quad (7)$$

where V is the volume of the worn profiles being removed, s is the sliding distance between the contact surfaces, K is the dimensionless wear coefficient, W is the normal load and H is the surface hardness of the profiles. For the case of meshing gears, the expression takes the following form:

$$\frac{h}{s} = kp \quad (8)$$

where p is the Hertzian pressure at the contact point, k is the dimensional wear coefficient and h is the wear depth that needs to be minimized.

For a given contact point P, the wear depth h_p can be calculated as:

$$h_p = \int_0^s kp \, ds \quad (9)$$

The wear depth of P after n meshing cycles can be determined by the following expression

$$h_{p,n} = h_{p,n-1} + kp_{p,n-1}s_p \quad (10)$$

where $h_{p,n}$ the wear depth after n meshing circles, $h_{p,n-1}$ and $p_{p,n-1}$ the wear depth and the contact pressure respectively after $n-1$ meshing cycles and s_p the relative sliding distance of the contact point P

The sliding distance of the points P₁ and P₂ of the pinion and the gear respectively that corresponds to the contact point P can be calculated as

$$s_{p1} = 2a_H \left(\frac{V_1 - V_2}{V_1} \right) \quad (11)$$

$$s_{p2} = 2a_H \left(\frac{V_1 - V_2}{V_2} \right) \quad (12)$$

where a_H is the hertzian half-width and V_1, V_2 are the sliding velocities of P₁ and P₂ respectively.

After combining the above equations, the wear depth of the contact point P after N meshing cycles can be determined as:

$$h_p = Nkps_p \quad (13)$$

The hertzian half-width can be calculated by the following relations:

$$\alpha_H = \sqrt{\frac{4F_t R^*}{\pi E^*}} \quad (14)$$

$$F_t = \frac{2T_1}{d_{b1}b} \quad (15)$$

$$R^* = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \quad (16)$$

$$E^* = \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)^{-1} \quad (17)$$

where F_t the circumferential load on the tooth flanks, T_{out} the output torque, $d_{0.2}$ the gear's pitch circle, b the gear wheels' width, R^* the relative radius of curvature at each contact point, R_1, R_2 the radius of curvature of the pinion and the gear respectively at each contact point, E^* the equivalent modulus of elasticity, E_1, E_2 the modulus of elasticity of the pinion and the gear respectively and ν_1, ν_2 the Poisson ratio of the pinion and the gear respectively.

To accurately model the load sharing process between meshing teeth without introducing a time-consuming numerical process into the optimization algorithm, the following approach was implemented. After calculating the highest and lowest points of single tooth contact for each pair of conjugate profiles, the entire load is applied to each tooth flank during the single-tooth contact period. However, during the double-tooth contact, the load is reduced as it is distributed among multiple pairs of teeth. To ensure a smooth distribution of wear depth

across the tooth flank, the load-sharing model presented in Fig. 5 was incorporated.

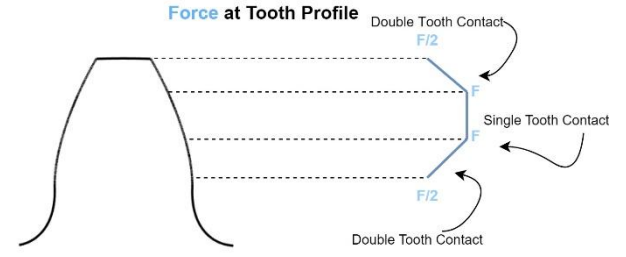


Fig. 5. Load distribution model on tooth flank profile.

2.4 Optimization algorithm

The majority of published studies, particularly in the last two decades, focused on gear optimization have employed evolutionary algorithms due to their capability to effectively avoid local minima and their relatively lower computational cost compared to earlier approaches [1]. Among the various evolutionary algorithms, the genetic algorithm [32] is the most widely used.

For this optimization problem, the genetic algorithm (ga) incorporated in MATLAB was utilized. To obtain the best possible results, the algorithm's parameters were systematically investigated. The tournament selection method was chosen to determine the 'parents' for the subsequent generations in the optimization process. The mutation function employed was 'mutationadaptfeasible' since the optimization variables were subject to constraints. Additionally, due to the presence of constraints in the optimization problem, the 'crossoverintermediate' was selected as the crossover function. The specific values for the crossover fraction, elitism, maximum generations, and population data were independently determined for each individual case.

Another significant parameter for the optimization algorithm is the number of variables, which is directly related to the number of control points. More specifically, each coordinate of the control points serves as an optimization variable, except for two cases; one of the coordinates of the first and last control points is determined directly to ensure that the lower and upper radii of the tooth flank are fixed.

As for the number of the contact points, apart from the fact that a high number of control points and thus variables could lead to convergence issues for the algorithm, it was found that as the number of control points increases, the number of the produced curves that actually adhere to the constraints reduces significantly. For the cases of 5 and 7 control points, 10,000 free-form curves were produced for each case and tested to determine if they can firstly comply to the law of gearing and if yes then if they have cusps/discontinuities or not. As it can be seen by the results the percentage of the free-form curves that are valid tooth flank profiles falls from 30.9% for the case of 5 control points to only 2.5% for 7 control points.

Regarding the number of contact points, it was observed that a high number of control points and variables could not only potentially lead to convergence issues for the optimization algorithm, but it also results in a significant

reduction in the proportion of generated curves that actually comply with the constraints. To investigate this, 10,000 free-form curves were generated using 5 control points and another 10,000 using 7 control points and were tested for compliance with the law of gearing and the presence of cusps or discontinuities. The results (Table 1) indicate that the percentage of valid tooth flank profiles decreases from 30.9% for 5 control points to only 2.5% for 7 control points. Consequently, the exact number of control points should be also selected for each case independently.

Table 1. Distribution of profile curves regarding their adherence to the law of gearing and the geometrical constraints for the case of 5 and 7 control points (10,000 profiles each).

	Do not comply to law of gearing	Contain cusps	Valid profiles
5 control points	37.9%	31.2%	30.9%
7 control points	72.9%	24.6%	2.5%

3 Results

The optimization process was conducted on two distinct case studies, as outlined in Table 2. The first case study was chosen based on the specifications of the FZG test rig implemented for the experimental evaluation of the optimized gears. A second case study that differs significantly in terms of the transmission ratio was examined to allow for a comprehensive assessment of the optimization procedure's effectiveness under varying conditions.

Table 2. Geometrical parameters of case studies

	Case study #1	Case study #2
Number of teeth - pinion	30	19
Number of teeth - gear	31	49
Transmission ratio	1.033	2.579
Module (mm)	3	3
Pressure angle (deg)	20	20
Width (mm)	45	17

3.1 Case study #1

The first case study involves a spur gear pair with a transmission ratio of 1.033, which is utilized in the FZG testing machine described in Section 4. To achieve the best possible outcome, the optimization settings were parametrically explored. Various numbers of control points were employed, and different values for optimization settings such as crossover fraction and population size were tested. The geometric and operational parameters utilized in this case study are summarized in Table 3, while the optimization settings employed are listed in Table 4.

Table 3. Geometrical, material and operational parameters of case study # 1

Parameter	Standard involute gears	Optimized non-involute gears
Number of teeth - pinion	30	
Number of teeth - gear	31	
Transmission ratio	1.033	
Module (mm)	3	
Width (mm)	45	
Addendum coefficient (mm)	1.0	0.85
Dedendum coefficient (mm)	1.25	1.0
Output Torque (Nm)	10	
Input speed (rpm)	1,000	
Operation time (min)	180	
Material	PLA	
Modulus of elasticity (MPa)	680	
Poisson ratio	0.35	
Wear coefficient (mm ² /N)	10 ⁻⁷	

Table 4. Optimization parameters of case study #1.

Setting	Value
Crossover fraction	0.4
Population size	200
Maximum generations	200
EliteCount	20

A range of control points from 4 to 7 was implemented in the optimization process, and the results are presented in Table 5. The corresponding generated profiles can be observed in Fig. 6. Two objective functions were considered; the first one focused solely on the integral of the wear depth along the contact path of the meshing tooth flanks (Eq. 18), while the second one incorporated the maximum absolute values of the wear depth in addition to the integral (Eq. 19). Unless specified otherwise, the second objective function was employed for the optimization processes as it yielded slightly improved and notably smoother wear depth distributions.

$$F_{obj1} = \left| \int h_{p_{pinion}}^2 dx \right| + \left| \int h_{p_{conjugate}}^2 dx \right| \quad (18)$$

$$F_{obj2} = \left(\left| \int h_{p_{pinion}}^2 dx \right| + \left| \int h_{p_{conjugate}}^2 dx \right| \right) + 0.04 \max \left(\left| h_{p_{pinion}} \right| + \left| h_{p_{conjugate}} \right| \right) \quad (19)$$

As mentioned in Section 2.2, the addendum and dedendum coefficients of the optimized gears were adjusted. As depicted in Fig. 7, the optimized profiles exhibit a steeper slope along their length, which raises

undercutting issues. To address this, both the addendum and dedendum coefficients are reduced, as indicated in Table 3.

The results presented in Table 5 and Fig. 8 demonstrate that the generated non-involute profiles exhibit significantly improved wear resistance compared to their standard involute counterparts. The average wear depth was reduced by up to 53.6%, while the maximum wear depth was reduced by up to 69.1%. The reduction in the addendum and dedendum coefficients played a role in both the average wear depth reduction, as it decreased the length of the contact path, and the maximum wear depth reduction, as the wear mechanism is more active at the extremities of the tooth profiles. However, this reduction in coefficients also led to a slight decrease in the already -anticipatedly- reduced contact ratio, which is slightly below 1.2 for all cases.

Even though all the generated non-involute curves demonstrate improved wear performance, those created from a higher number of control points exhibit a more wavy profile. While wavy profiles have the potential to yield even better results, their practical implementation is often challenging due to their sensitivity to assembling errors, such as center distance variations. Therefore, the smoother non-involute profiles generated with 4 control points (Fig. 6) are considered the most suitable for this application, as they also offer the best results in terms of average wear depth and exhibit the higher contact ratio.

Table 5. Optimization results for case study #1.

	Involute	Optimized non-involute			
Number of control points	-	4	5	6	7
Polynomial order	-	3	4	4	4
Contact ratio	1.67	1.17	1.13	1.15	1.13
Average wear depth – pinion (μm)	11.1	5.6	5.8	6.5	6.5
Average wear depth – gear (μm)	11.0	5.1	5.6	6.2	6.1
Max. wear depth – pinion (μm)	33.3	10.8	10.3	11.0	10.9
Max. wear depth -gear (μm)	32.7	15.4	13.1	14.4	15.4

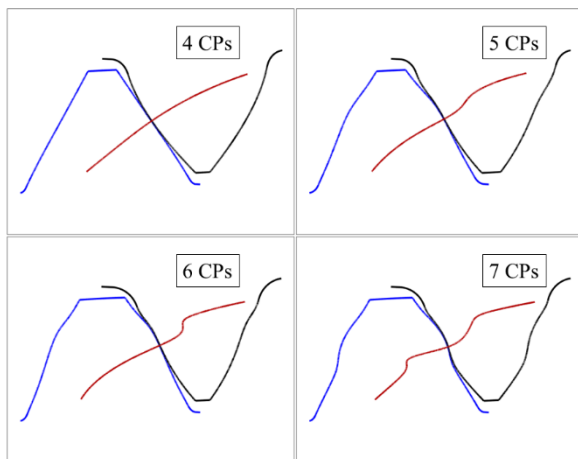


Fig. 6. Optimized free-form tooth flanks (blue) with their conjugate profiles (black) and the resulting contact paths (red) implementing 4, 5, 6 and 7 control points for case study #1.

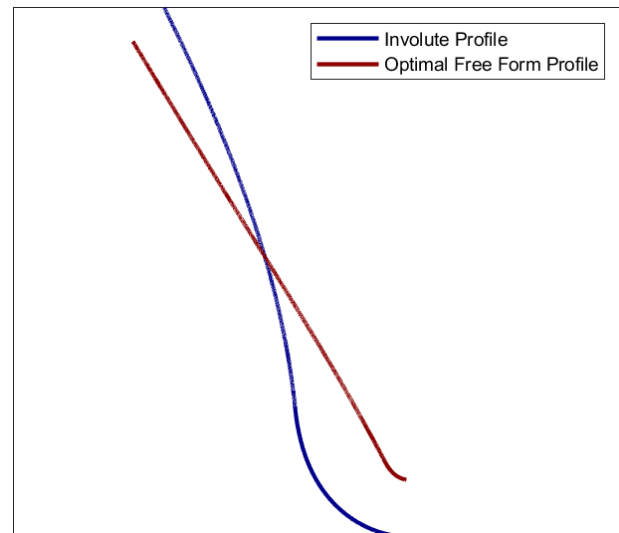


Fig. 7. Comparison of optimized free-form pinion profile (red – 4 control points) with its standard involute counterpart (blue) for case study #1.

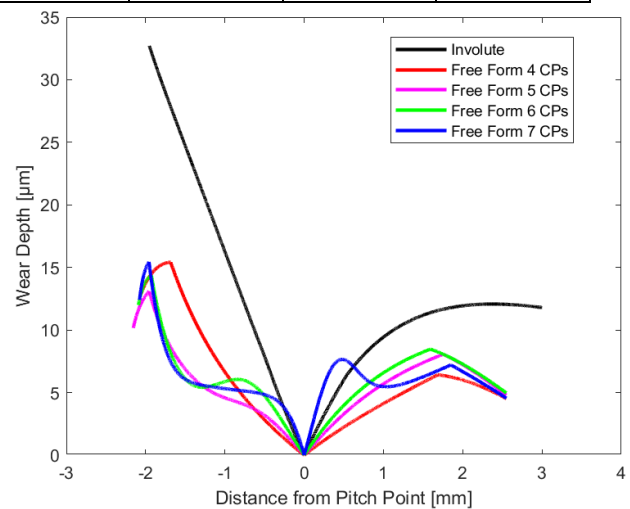


Fig. 8. Comparison of wear depth along the tooth flanks of the free-form gears generated with 4, 5, 6 and 7 control points and their standard involute counterpart for case study #2.

3.2 Case study #2

For the second case study, a gear pair with a transmission ratio of 2.579 was employed. The material of the gears remained the same as in the previous case study, while the operational characteristics, specifically the input speed and the output torque, were altered as outlined in Table 6. Slight variations in the optimization settings were implemented to obtain the final results, as presented in Table 7.

In this case study, even greater performance improvements were achieved. The average wear resistance was reduced by up to 56.6%, and the maximum wear depth was reduced by as much as 79.7%. The non-involute curves generated from different numbers of control points, as shown in Fig. 9, exhibit remarkable similarity, demonstrating the robustness of the optimization process. Among these curves, the one generated from 5 control points was selected due to its slightly higher contact ratio value.

Parameter	Standard involute gears	Optimized non-involute gears
Number of teeth - pinion	19	
Number of teeth - gear	49	
Transmission error	2.579	
Module (mm)	3	
Width (mm)	17	
Addendum coefficient (mm)	1.0	0.8
Dedendum coefficient (mm)	1.25	1.0
Output Torque (Nm)	2.5	
Input speed (rpm)	1,500	
Operation time (min)	180	
Material	PLA	
Modulus of elasticity (MPa)	680	
Poisson ratio	0.35	
Wear coefficient (mm ² /N)	10 ⁻⁷	

Table 7. Optimization parameters of case study #2.

Setting	Value
Crossover fraction	0.6
Population size	200
Maximum generations	200
EliteCount	10

Table 6. Geometrical, material and operational parameters of case study #2.

Table 8. Optimization results of case study #2.

	Involute	Optimized non-involute			
Number of control points	-	4	5	6	7
Polynomial order	-	3	4	4	4
Contact ratio	1.64	1.19	1.23	1.20	1.10
Average wear depth – pinion (μm)	19.8	8.6	8.6	9.4	11.5
Average wear depth – gear (μm)	23.9	9.3	9.7	10.7	12.2
Max. wear depth – pinion (μm)	157.4	32.0	42.0	37.2	35.2
Max. wear depth -gear (μm)	52.2	20.8	19.8	22.9	25.8

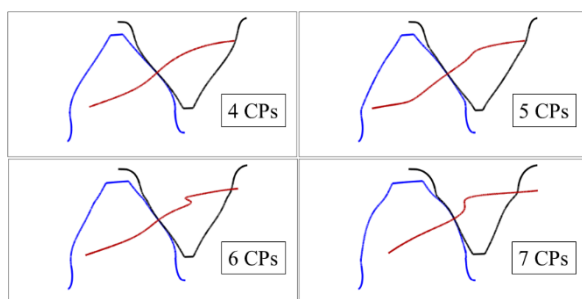


Fig. 9. Optimized free-form tooth flanks (blue) with their conjugate profiles (black) and the resulting contact paths (red) implementing 4, 5, 6 and 7 control points for case study #2.

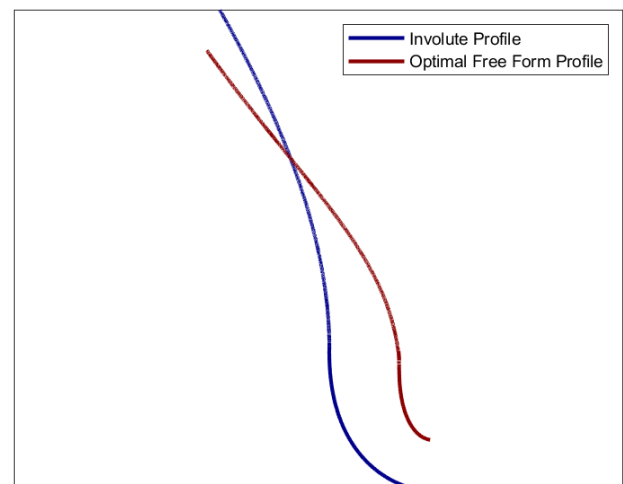


Fig. 10. Comparison of optimized free-form pinion profile (red – 5 control points) with its standard involute counterpart (blue) for case study #2.

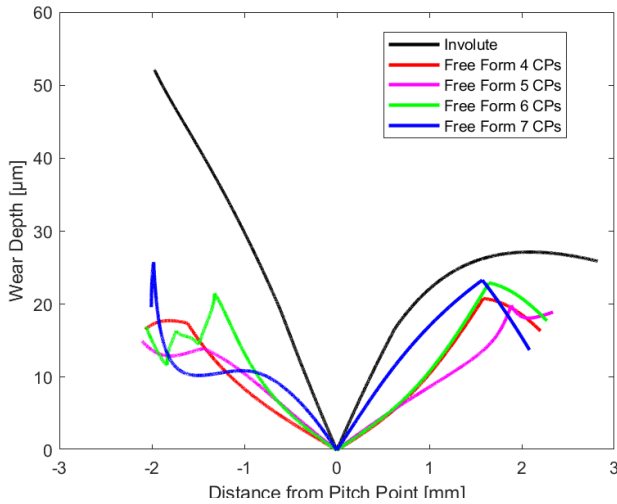


Fig. 11. Comparison of wear depth along the tooth flanks of the free-form gears generated with 4, 5, 6 and 7 control points and their standard involute counterpart for case study #2.

4 Experimental validation

All spur gears were manufactured through Fused Deposition Modeling (FDM) using a Creality Ender CR10-S5 printer, which was chosen for its large bed, allowing both gears to be printed simultaneously. STL files were generated from the original CAD models, and the final G-code was created using CURA Slicer Software, with the printing properties specified in Table 9. These parameters were selected after a large number of trials to achieve the best possible accuracy for the tooth flank profile. The produced gears are illustrated in Fig. 12.

Table 9. Printing parameters.

3D Printer Model	Creality Ender CR10-S5
Printing method	FDM
Hotend Temperature [°C]	200
Bed Temperature [°C]	50
Layer Height [mm]	0.16
Infill Density [%]	80
Printing Speed [mm/s]	25



Fig. 12. Optimized free-form (up) and standard involute (down) spur gear pairs produced with FDM.

In order to evaluate the main form errors of the gear teeth in relation to the nominal CAD model, a Coordinate Measuring Machine (CMM) is utilized (Fig. 13). The CMM model and its specifications are shown in Table 10. The process begins with importing the CAD model of the gear into the graphic environment of PC-DMIS. Additionally, a point cloud is imported into the PC-DMIS software to guide the CMM. A total of 50 points are used on each side of the tooth, positioned at a specific vertical distance from the upper surface of the gear.

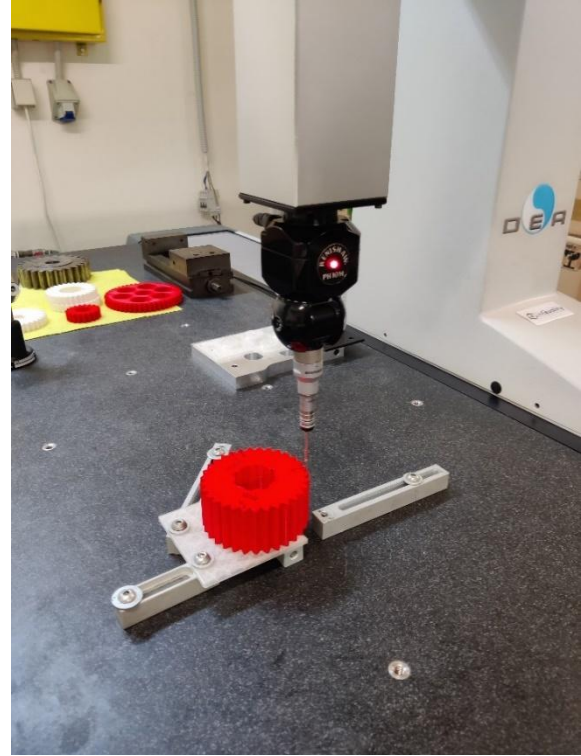


Fig. 13. Gear measurement on CMM.

Table 10. CMM specifications.

CMM Model	DEA Global 071005
Measuring Dimensions (L x W x H) [mm]	700 x 1000 x 500
Accuracy [μm]	± 6
Prehit Distance [μm]	0.8
Retract Distance [μm]	0.8
Probe Diameter [mm]	1

Once the experiments at the test rig are completed, the gears are measured again to calculate the final wear. Using the same PC-DMIS guide program, precise measurements of the final wear on the gears are ensured. To enhance repeatability in measurements, jigs are designed and 3D printed for each gear (Fig. 14). This approach eliminates the need for realignment after completing the measurements, as it matches the setup done before. The measurement data is exported as text files and then imported into MATLAB software to calculate the wear depth.

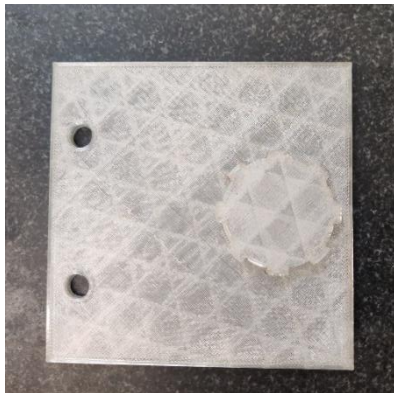


Fig. 14. Gear mounting jig for CMM Measurement.

For the experimental evaluation of Case Study #1, the FZG test rig illustrated in Fig. 15 is used. Both standard involute and optimized free-form spur gears are tested for comparison. As stated before, the optimized profile generated by 4 control points was selected as the free-form tooth flank. Following the procedure described above, the gears are manufactured and then measured by the CMM to obtain the initial profile of the tooth flanks. Subsequently, the spur gears are subjected to a load of 10 Nm for 180 minutes using the FZG test rig. Afterward, they are measured again in the CMM machine and the wear depth at each point is determined.



Fig. 15. FZG test rig.

Some representative results are presented in Fig. 16. Firstly, it is evident that the distribution of the wear depth along the tooth flank does not precisely follow the theoretical distribution presented in Figs. 8 and 11. This discrepancy is mainly due to the geometrical deviations of the tooth flanks caused by 3D printing accuracy. Although the achieved accuracy was not poor for 3D printed parts, the resulting profiles exhibited a wavy pattern and the wear depth distribution followed this profile as illustrated in Fig. 17 (right). Geometrical deviations were more pronounced in some teeth than others, as depicted in Fig. 17 (left), resulting in local increases in wear depth, such as in the tip regions of teeth #2 and #20 of the free-form conjugate and the involute pinion respectively (last two diagrams in Fig. 16). However, the majority of the tooth flanks, despite containing geometrical deviations, yielded representative wear depth distributions that allowed for a valid comparison between the free-form and the involute profiles.

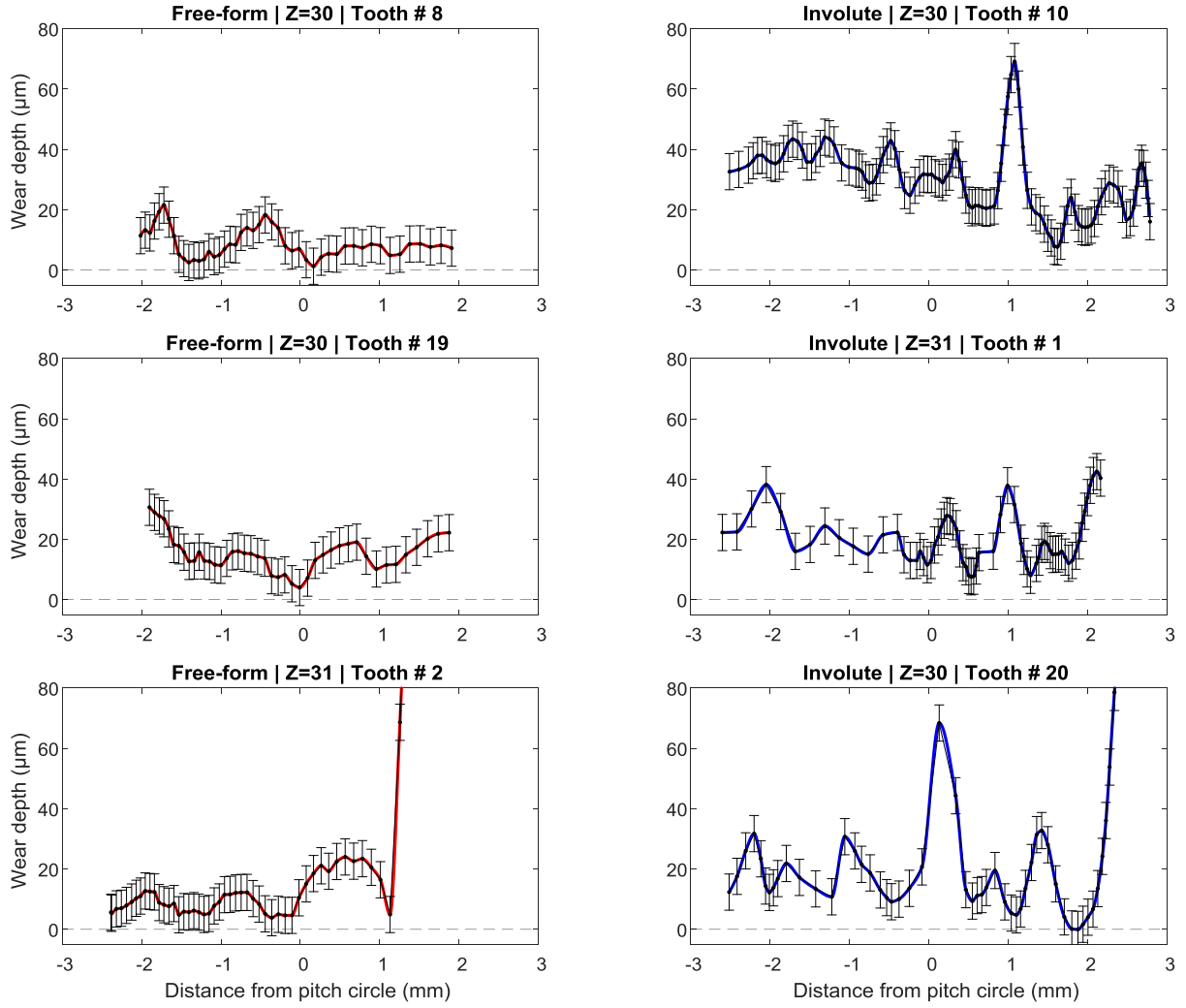


Fig. 16. Wear depth of optimized free-form (left) and standard involute (right) spur gears

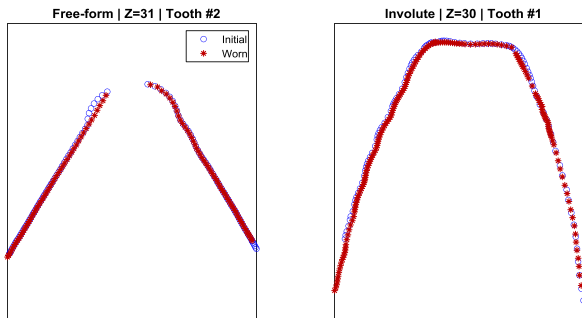


Fig. 17. Initial (blue) and worn (red) profiles of two representative teeth containing geometrical deviations due to 3D printing inaccuracies

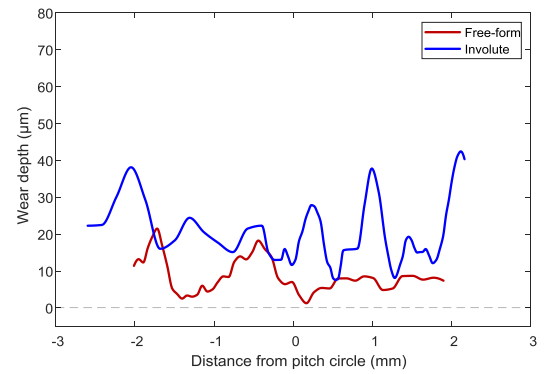


Fig. 18. Wear depth distribution of optimized free-form (red) and standard involute (blue) spur gears

Table 11. Experimental results regarding the average wear depth of standard involute and optimizes free-form gears.

Gears	Average wear depth (μm)
Standard involute	32.0
Free-form	17.9

5 Conclusions

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In this study, a comprehensive free-form optimization process was conducted to determine the optimal tooth profile that minimizes wear on 3D printed spur gears during operation. The tooth flank geometry was represented using a B-spline curve, and a genetic algorithm was employed to determine the best positions for the control points, aiming to minimize wear depth across the tooth flanks. The major outcomes of the study are summarized below:

- Development of a robust and comprehensive free-form tooth flank optimization algorithm capable of handling the complex geometric constraints of tooth profiles and generating conjugate flanks with improved performance.
- Implementation of two case studies to minimize the wear depth of spur gears manufactured from PLA through 3D printing. In these cases, the optimized free-form gears achieved a significant decrease in average wear depth, with reductions of 53.6% and 56.6% compared to their standard involute counterparts. Additionally, the maximum wear depth was reduced by 69.1% and 79.7% respectively.
- Conducted experiments using an FZG test rig and performed measurements on a CMM both before and after testing. The results showed that the optimized free-form gears experienced 44.1% less wear than their standard involute counterparts.

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International collaboration: benefits and contributions for scientific work

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Abstract. The Erasmus program is a European education and mobility initiative that provides exchange opportunities for students, PhD students, teachers and staff between different universities and educational institutions in Europe. Through the Erasmus program, students have the opportunity to enrich their knowledge and gain deeper knowledge in a field that interests them. In addition, they have the opportunity to spend a period of their education in another country where they can enjoy a new culture, language and educational system.

Keywords: ERASMUS PROGRAM, TRAINING, EDUCATION, EDUCATIONAL SYSTEM

1 Introduction

The Erasmus program is a European educational and mobility initiative that provides opportunities for student, faculty, and staff exchanges between different universities and educational institutions across Europe.

Through the Erasmus program, students have the chance to spend a certain period of their education in another country, where they can experience a new culture, language, and educational system.

The program supports the academic, cultural, and professional development of participants by offering them diverse perspectives and challenges. It encourages international cooperation and knowledge exchange, fostering closer ties between various European educational institutions.

The exchange of knowledge and experience through Erasmus enriches the personal development of participants and helps them become more adaptive and open to global challenges.

1.1 University of Pitești, Romania

As a doctoral student at Ruse University, you had the opportunity to participate in the Erasmus mobility program. Your Erasmus program started in 2019, and your first mobility was in Pitești, a city in our neighboring country, Romania.

The University of Pitești attracts the interest of many young people due to its excellent academic programs and the high-quality education provided by its exceptional academic staff. The university's mission encompasses education, scientific research, and public services, achieving the trinomial of a modern university.



Pic. 1: University of Pitești

The University of Pitești generates and disseminates knowledge through fundamental and applied scientific research, development, innovation, technology transfer, individual and collective creativity, all of which hold significance at regional, national, and international levels.

Associate Professor Dr. Eng., the Erasmus+ Office Coordinator at the University of Pitești, introduced us to faculty members from various departments and faculties, as well as colleagues from other departments. Throughout the mobility, you engaged in various activities and experiences, such as expanding your knowledge and skills in working with CAD systems, particularly focusing on SOLIDWORKS.

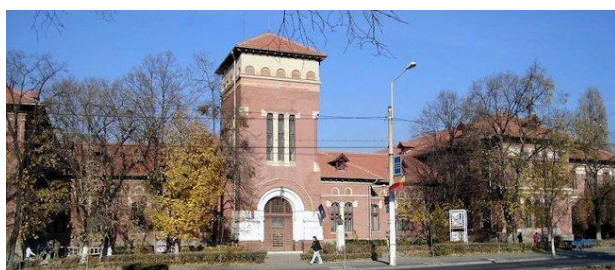
Near Pitești there is a factory for the production of "Dacia" cars. Several auto parts manufacturers are positioned in the region itself. During your mobility, you went through several interconnected stages, including meetings with university representatives, organizers of the university's annual scientific conference and fellow PhD students. Associate Professor Dr. Eng. Erasmus+ office coordinator at the University of Pitești, connected us with professors from various departments and faculties. The University of Pitești cooperates actively with local authorities to solve the city's problems related

to transport and the road network. In their article "Opportunities to improve road traffic on short-length road connections by correlating intersection traffic lights", A Boroiu, E Neagu and A A Boroiu [1]. Authors of the publication, consider possible solutions to improve road traffic in the central area of cities, for which is a characteristic longitudinal arrangement of the street network, with application to the case of Piteshti, where the road network in the central area consists of two main road networks located longitudinally, with one-way regulated traffic, interconnected by several streets. Through a detailed analysis of road traffic, the authors of the paper found that a specific problem of a road network with very close main arteries that are arranged in parallel and connected by very short road links is the insufficient capacity of these connecting arteries for traffic flows generated in important network nodes. For the case where these road junctions have a regulation regulated by traffic lights, the easiest solution may be to correlate the green phases of these junctions so that the connecting artery does not create queues that would exceed the storage capacity of that artery, limited mainly to its very short length.

1.2 The University of Craiova, Romania

The University of Craiova, Romania, offers another opportunity for mobility through which students and doctoral candidates can enhance international collaboration and exchange knowledge and experience, fostering closer ties between various European educational institutions.

The Faculty of Mechanics at the University of Craiova was founded in 1977 and offers specializations similar to those provided by the University of Ruse, such as Mechanical Engineering Technology and Metal Cutting Machines. Continuing the tradition of higher education in the technical field, in 1990, the University of Craiova introduced two additional specializations: Road Vehicles and Machines and Installations for Agriculture.



Pic. 3: Faculty of Mechanical Engineering

Similar to the previous mobility experience, we expanded our knowledge and skills in working with CAD systems, with a specific focus on SOLIDWORKS. During our mobility in Craiova, our gracious host, Professor Dr. Eng. Nicolae Krachunu, introduced us to all the educational facilities and laboratories in the Faculty of Mechanics. We paid particular attention to the

article by N. Dumitru, E. Dragut, N. Craciunoiu, and I. Geonea titled "Experimental Stand for Cylindrical Gears. Efficiency Measurement" [1], where a testing stand for investigating the efficiency of cylindrical gears is discussed. The authors of the article built a virtual model of the testing stand, which they use for testing both as a virtual model and a manufacturing model. The model was developed in SolidWorks.

The main principle of the testing stand is that loading is performed internally within a closed loop composed of two sets of cylindrical gears, eliminating the need for a braking system. The author team used an HBM system for data collection and strain gauge transducers to determine the experimental torque variation on the gear shafts. This way, the experimental efficiency of the set of cylindrical gears is established.

The stand is designed in a modular design. A variety of gear pairs with the same axis distance can be used for testing.

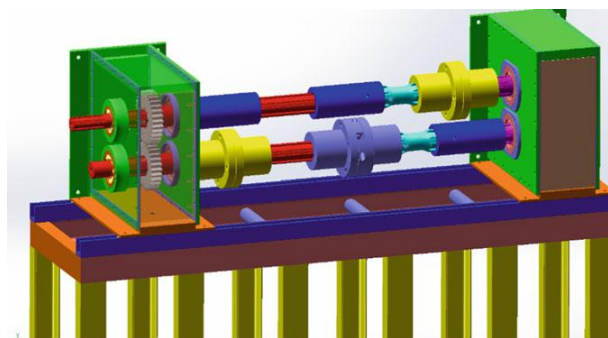


Fig. 1 Design model of the test bench for spur gears efficiency, [2].

Getting a closer look at this and some other articles by our Romanian colleagues provided us with the opportunity to expand your knowledge and experience in the field of mechanics and technologies, while also introducing us to new educational and research resources that contribute to enriching our academic and scientific journey.

During this mobility, we visited the technical facilities and laboratories at the Faculty of Mechanics. We had meetings with the university's management, organizers of the scientific conference, and professors from various departments, as well as colleagues.

Part of our tour included a visit to the university's library, where we discussed opportunities for research and collaboration with faculty from the department and the university.

Overall, the second mobility was just as enriching and valuable as the first, providing us with valuable insights and experiences in engineering and academia.

Based on the meetings with colleagues and professors at the University of Craiova, Romania, long-lasting and fruitful professional relationships were established.

In addition, we visited a branch of the University, which is located in Drobeta Turnu Severin. As there is a large shipyard near Turnu Severin where both sea and river vessels are built, the main focus of the faculty's activity is related to this type of research.



Pic. 4: Drobeta Turnu Severin

The city is well-landscaped and clean, offering a balanced experience between a serene and relaxing environment and a city with various services and opportunities. I believe it is one of the best cities in Romania and Eastern Europe. I highly recommend both the city and the University of Craiova.

1.2 Conclusions

In conclusion, I can summarize that the Erasmus program facilitates the improvement and advancement of knowledge. It leads to the acquisition of experience, which is highly beneficial not only for the student, doctoral candidate, or educator, but also for the entire student community.

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Investigation of selected materials stability for future application in development of small fast modular reactors (SFMR)

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Abstract. Currently, there is growing demand for cheap and reliable electricity production. One possible solution could be the construction of small modular fast neutrons nuclear reactors, where the reactor core cooling fluid is a suitable liquid metal. Liquid tin has significant advantages over the other potential candidates but its usage as coolant is challenging for the confining materials ensuring the nuclear safety of the future reactors. Current paper presents an investigation of structural materials stability in liquid tin with measurement of the surface tension, based on "Wilhelmy plate" method. The results are related to stability of spring steel, iron, nickel and copper, used as dipping plates as well as oxygen/liquid tin interactions when melted tin is exposed to air. The reported custom apparatus in our previous studies, was improved and its measuring capability was enhanced in order to demonstrate reliable results at low cost. Theoretical analysis is provided in order to demonstrate the reliability of the presented approach. Main results include: (1) Determination of liquid tin surface tension in case of air oxygen/tin reaction in liquid surface; (2) Investigation of measuring plate degradation and calculation of solution rate constant in case of dipped a plate made by copper; (3) Determination of Butler's equation parameters in case of surface oxygen/tin interactions.

1 Introduction

In recent years Small Modular Fast Reactors (SMFR) have attracted significant attention and several design projects were launched. One of the major problems appears in substantiation of the coolant that in many cases is liquid metal. In the current studies, we turned our attention to the possible application of liquid tin (Sn), (melting point: 505 K). It has several advantages over the other alternatives, [1]. The investigations of liquid tin were initiated during 1950s. One of the earliest proposals for application of liquid tin as a coolant, was published by Weeks in 1971, [2,3]. Besides, liquid tin is widely used as a solder in electronic industry for decades, having relatively well-known properties. The most severe disadvantage is its aggressive interactions with structural materials, currently used in nuclear technology. However, it could still be compatible with graphite (C), Tungsten (W), Molybdenum (Mo), Rhenium (Re), below 773 K with Chromium (Cr) and with Iron (Fe), [3]. Current paper presents our approach for investigation of liquid tin surface interactions with several materials such as iron (Fe), Spring steel (SS), nickel (Ni) and copper (Cu). Chapter two represents our theoretical approach, where a simplified analytical model is presented, based on Butler equation and solution rate constant of copper plate is estimated. The experimental approach, materials and used setup are presented in chapter three. Chapter four

provides the experimental results and analysis of the obtained data. An approach for material dissolution in liquid tin estimation is also presented. Finally, chapter five summarizes the results and outlines the directions for future studies.

2 Theoretical basics

In order to analyse oxygen/tin interactions at the liquid surface, we need to study their equilibrium diagram as well as their chemical reactions. The results from the analysis indicate that determination of surface tension may play significant role in these studies.

2.1 Oxygen-tin equilibrium diagram

Solid tin exists in three polymorphic forms which are stable at the corresponding thermal conditions. At temperatures below 286.35 K stable "α" tin (grey) is formed. In temperature range between 286.35 K and 434.15 K, tin exists in stable "β" form (white). Above this range, white tin is transformed to "γ" form which melts at 505.05 K, Fig. 1. Gray tin crystallizes in structure similar to the diamond and has semiconducting properties, [4]. The white tin crystallizes in tetragonal structure and behaves as a typical metal, [4]. The "α" phase of tin crystallizes in rhombic crystals. The two forms really differ in their densities and this way, the transition from white to gray form is marked by

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significant change of tin volume, [4]. The equilibrium diagram in Fig. 1 is made, based on the calculations performed in [5,6]. From the diagram, one is seen that in the range between 508.05 K and 543.4 K, at the atomic percentage of oxygen content below 50% coexist two phases liquid “ γ ” tin and solid SnO.

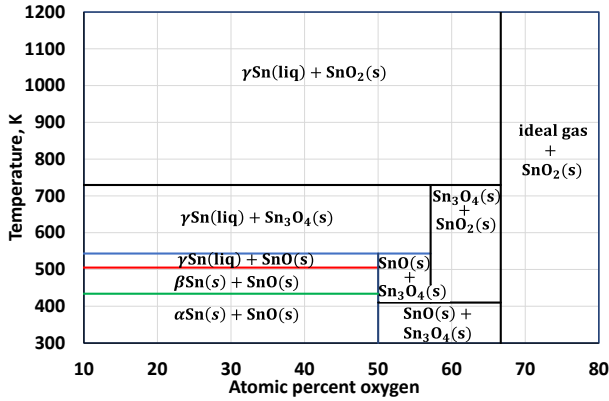


Fig. 1. Constructed O-Sn equilibrium diagram [5].

Above 543.4 K, a transition from SnO to Sn₃O₄ takes place, which can significantly affect the quality of the surface tension measurements in case of liquid tin oxidation (see Fig. 1).

2.2 Surface interactions

Since all metal oxides by their nature are ionic, the transport of neutral metals across the oxide interface is not feasible. However, transport of ions is well explained and in the case of Frenkel defects, ionic vacancies are transported mainly by cation vacancies, where cations are free to migrate any direction of cations, [7].

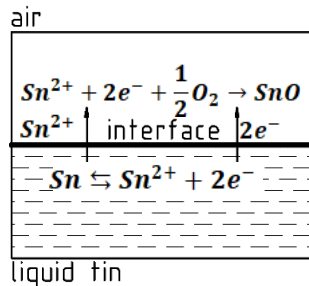


Fig. 2. Interfacial reactions and transport with cation mobility.

According to Ref. [8], the surface oxidation of liquid tin passes through five steps (see Fig. 2.): (1) Sn atoms reduce as Sn²⁺ cations at the liquid interface; (2) electrons diffuse upward to reach outer layer; (3) O₂ gas molecules adsorb the diffused electron and form O²⁻ anions on the oxide surface; (4) Two ions react and form SnO solid phase at the interface; (5) References [8,9], reported, based on experiment, that initially only SnO solid phase is formed that coexists at the surface with liquid tin. Further, with increasing of the oxygen content in liquid layer, SnO₂ is formed. However, from the diagram, (see Fig. 1), it can be seen that a solid Sn₃O₄ phase formation takes place, and it coexists at the surface with SnO for temperatures below 543 K and oxygen

content less than 50 at. %. Above this oxygen level, SnO₂ is formed, instead of SnO, Fig. 1. Note that above 50 at. % oxygen content, no liquid tin is present in surface layer. It is expected, tin oxidation at the surface to be suppressed. However, because of density difference between SnO and SnO₂, (about 5%), cracks and pores formation will be enabled at the surface that will enable diffusion of O²⁻ through the surface layer and further continuation of the oxidation process, [8,9].

2.3 Determination of surface tension

The exposure of the liquid tin surface at the atmospheric oxygen initiates immediately its oxidation process, leading to decrease of the surface tension, [10]. In Ref. [10] surface tension of 598 mN/m was measured at 560 K by the sessile drop method. In this case, no oxygen is present during the measurements and consequently the obtained results are for a clean liquid surface without oxidation, while for a surface rich in oxygen it was found to decrease down to 535 mN/m. In our studies, the surface tension was measured by “Wilhelmy plate” method and at the time of dipping, the oxidation is already an ongoing process. To solve this problem, we propose the following procedure: clean the liquid tin surface carefully and begin tracking the time just after. Recorded, time dependent surface tension, $\sigma(t)$, (in N/m) evolves with time until stable readings are obtained. The following equation is then applied to the obtained data:

$$\Gamma(t) = \Gamma_0(T) \exp \left[\frac{\sigma(t)}{b(T)} \right] \quad (1)$$

where the constant $\Gamma_0(T)$ is arbitrary for any particular temperature, T . Determination of constant $b(T)$ can be done in the first 200-300 seconds where time dependent measurements, shows sloped linear behavior. This is not surprising, since for time $t \rightarrow \delta t \rightarrow 0$, Eq. (1) can be expanded in a Maclaurin series:

$$\Gamma(\delta t) \approx \Gamma_0(T) \exp \left[\frac{\sigma(0)}{b(T)} \right] \left\{ 1 + \frac{\sigma'(0)}{b(T)} \delta t \right\} \quad (2)$$

where $\sigma'(0)$ is the time-derivative at $t = 0$. This expansion can be rearranged in the form:

$$\Gamma(\delta t) \approx \Gamma_0(0) + \frac{\sigma'(0)\Gamma_0(0)}{b(T)} \delta t \quad (3)$$

In this relation the only adjustable parameter is $b(T)$. The other parameter $\Gamma_0(0)$ is of no importance for the linearization and we could select any arbitrary values (as mentioned above). For our calculation it was kept constant for all temperatures. Linearization procedure involves optimization of parameter $b(T)$ [in eq. (1)] for $\Gamma_0(T) = 1000000.0$, until an inclined line is obtained. During the optimization procedure, a polynomial interpolation was applied to find the optimal values of parameter $b(T)$. To achieve this goal, a custom-made Python script was developed, based on the data in Refs. [11,12]. The convergence in calculations takes place when the solution minimizes the squared error. Further,

the surface tension at $t = 0$ can be determined from Eq. (1) as follows:

$$\sigma(0) = b(T) \ln \left[\frac{r(0)}{r_0(T)} \right] \quad (4)$$

Correctly determination of the initial time, after the surface cleaning, is important and it can increase significantly the accuracy of the calculations.

Figure 3 schematically represents the dipping approach, where the plate, made of the selected material is inserted in such a way that the surface formed meniscus is attached to the plate's lower edge.

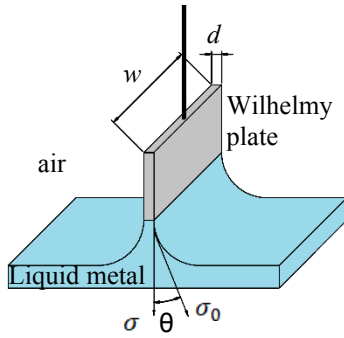


Fig. 3. The Wilhelmy plate dipped in molten metal.

Further, the plate is pulled in an appropriate way and the reaction force evolution with time is recorded. Therefore, the surface tension can be determined as:

$$\sigma(t) = \frac{F'}{P \cdot \cos(\theta)} \quad (5)$$

where $P = 2w + 2d$ is the contact perimeter of "Wilhelmy plate", (see Fig. 3); F' is the reaction of the pulling force and the angle θ indicates the surface tension vector direction regarding the normal to the surface. In case of complete wetting, $\cos \theta = 0$.

2.4 Study of material stability in liquid tin

Interaction of liquid tin with solid surfaces and their dissolution rates was investigated, based on the Shchukarev-Nernst equation, [13,14]:

$$\frac{dC}{dt} = K \frac{S}{V} (C_s - C) \quad (6)$$

After integration, for initial concentration of dissolved metal in the melt $C_0(t = 0) = 0$, following expression is obtained:

$$C = C_s \left[1 - \exp \left(K \frac{S}{V} t \right) \right] \quad (7)$$

where C_s is the concentration of saturated melt at the desired temperature, wt.%; S is the contact area of the surface, m^2 ; V is the volume of the melt, m^3 ; K is the solution rate constant, m/s , while C (wt.%) is concentration of dissolved metal at time t , s. In this case, it is important to be found from experimental data the values C_s for different temperatures. They are in the form:

$$C_s = A \exp \left(-\frac{B}{RT} \right), \text{ wt\%} \quad (8)$$

where, R is the universal gas constant in $JK^{-1}mol^{-1}$. The values of constants A and B are given in Table 1 for the investigated solid metals. The obtained data is expected to be used in estimation of solution rates constants for selected materials in our studies.

Table 1. Constants for saturated melt calculation

No.	Material	A	B	Ref.
1	Nickel	5.344	36794.1	[14]
2	Iron	24.374	27693.1	[15]
3	Copper	1708.378	34256.3	*

*Estimated, based on Sn-Cu equilibrium diagram in Ref. [16].

3 Materials, methods and experimental setup

A version of the experimental setup as well as the approach were reported previously in Ref. [1]. Since then, further development took place toward increase of obtained data precision and accuracy as presented below.

3.1 Method

The method itself consists of a plate ("Wilhelmy plate"), fabricated from proper corrosion resistant material, dipped into the liquid metal and pulled, in order to attach the meniscus on the lower plate perimeter, as shown in Fig. 3, where the surface reaction force to the pulling force is $F' = \sigma P \cos \theta$, [see Eq. (5)]. Proper further treatment of the plate allows contact angle between the plate and the surface menisci to be reduced to zero. In this case the equation reduces to $F' = \sigma P$. This way measurement is significantly simplified and the possible measurement errors are greatly reduced.

During higher temperature investigations (623 K), the "Wilhelmy plate" method shows relatively good stability. However, the surface tension measurement of metals poses some difficulties, related mostly to temperature gradients (around the liquid tin surface it is expected to be about 10 K) and chemical interactions with the oxygen in the air. Besides typical surface tension measurement, shown in Fig. 3, because the liquid surface is attached to the lower edge (perimeter) of the measuring plate, the force balance could create a dissolution front in case of plate dissolution. In this case, the solution of solid metal will take place only on that edge. This way, the solution rate constant (see sections 2.4&4.3) can become an important characteristic. Because of the liquid cohesion forces, despite the process of dissolution, there is expected the liquid surface to remain attached to the solid plate.

3.2 Experimental setup

The experimental setup shown in Fig. 4 as: a) sketched schematic and b) real picture. Heating bed is mounted on the movable z-axis (Fig. 4-2). The stage is confined in a box, where the atmosphere can be

controlled. The heater of the bed is isolated with a shell made of Polytetrafluoroethylene (PTFE). The temperature is controlled by an integrated Pt100 sensor and precise temperature controller that allows heating the sample up to 623 K. The sensor is also used for monitoring the temperature in real time. Position of the stage can be adjusted either manually or with the stepper motor (Fig. 4-8), controlled by a software. The supporting table and stand (Fig. 4-1,7) allows adjustment and levelling of the entire setup as well as easy installation of additional fixtures.

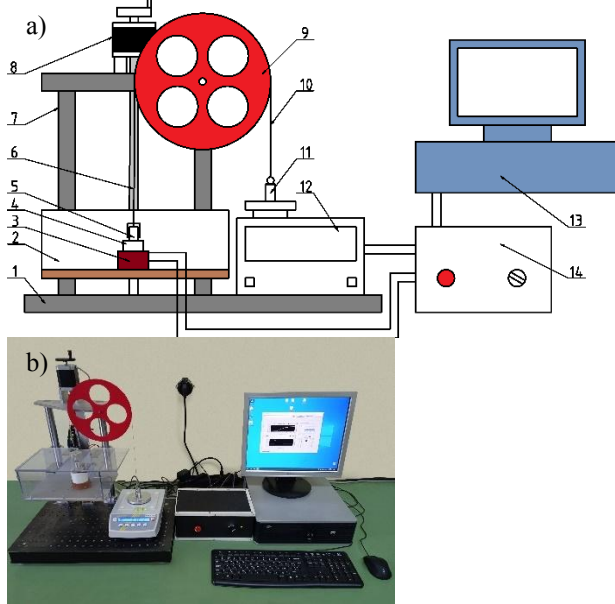


Fig. 4. Experimental setup: a) sketch, where: 1– supporting table; 2- isolated z-axis movable stage; 3 – heater; 4 – heating bed; 5 – Wilhelmy plate; 6 – non expanding string; 7 – supporting stand; 8 – stepper motor with manual control; 9 – pulley, 10 – non-expanding string; 11 – scale, 12 – data acquisition module, 13 – module for temperature control and position control, 14 – PC for data acquisition; b) picture.

The surface tension is measured by scale, KERN 240-KB-3N, that is calibrated before the beginning of measurements. A non-expanding string is attached to a weight, mounted on the scale and to the “Wilhelmy plate” holder. This string consists of two parts: thermally isolating tread and a stainless-steel chain (Fig. 4-6,10), both with very low weight. In order to initiate the measurement process, the plate is dipped in the liquid tin by moving the stage (Fig.4-2) upward until dipping takes place. Then, the stage is moved down until the lower edge of the plate, Fig. 4-5, attached on the liquid tin surface, reaches maximum pulling force, measured by scale, Fig. 4-11. Readings of the scales and the temperature controller are collected by the PC in real time, Fig. 4-14. A custom-made software solution, based on the “LabView” software is used for control and data acquisition process.

3.3 Materials

“Wilhelmy plates” made of four different materials are used for the tests (Table 2). The plates are prepared by laser cut metal sheets and their whiskers are cleaned

with sandpaper. In order to ensure zero contact angle between the plate and the liquid surface, they are coated with tin of the same grade as liquid in the heating bed (Chempur, CAS-No. [7440-31-5] R: 22-36/37/38-43 S:24026-37, Sn 99.5%). The zero-contact angle simplifies data processing and evaluation of the surface tension.

Table 2. Tested material as “Wilhelmy plates”

No.	Material	Content, [%]	Thickness, d [mm]	Width, w [mm]
1	Copper	99.7%	0,2	10
2	Nickel	99.3%	0,3	10
3	Iron	99.2%	0,1	10
4	Spring steel	AISI 1XXX*	0,15	10

*Fe 95.18%, Cu 2.76%, Si 0.63%, S 0.39 Mn, 0.26%, Cr 0.14%

3.4 The experimental procedure

Our experimental procedure includes the following steps, [1]: (1) cleaning selected plate and coating it with molten tin; (2) heating the oven with the tin sample. After reaching the desired temperature (for example, current maximum is 612 K), (3) the surface of liquid tin, exposed to air is carefully cleaned with a brush made of glass. At that point it is visually seen that formation of an oxide later starts immediately and the measurement of the time starts right after the cleaning. On the next step, (4), the dipping process is initiated in order attach the lower edge of the plate to the liquid surface (as described in section 3.1). We called this stage “Calibration of the measurement”. After dipping, a tension force opposing the surface meniscus force is applied by pulling the plate until the maximum reaction force is achieved. Then, (5), the measurement of the chemical reaction at the liquid surface begins, [1].

4 Results and discussion

The results of the investigation are in three directions: (1) identification of the surface tension of pure liquid tin; (2) solution rate constant determination in case of dipped plates dissolution; and (3) theoretical verification of Eq. (1).

4.1 Surface tension determination

Measurements of surface tension were performed, with plates made from nickel and iron. Obtained results, using the procedure, described in section 2.3, are presented in Fig. 5. The data are obtained after calculations, involving evolution of the surface tension after liquid tin was exposed to air. The data taken from Ref. [10], obtained from direct measurements of the surface tension of pure liquid tin, are used for a verification. Experimental results indicate that if iron plate is used for surface tension measurements, the overall trend of data (red line) predicts increasing of the

surface tension that contradicts with the data from Ref. [10], (green line).

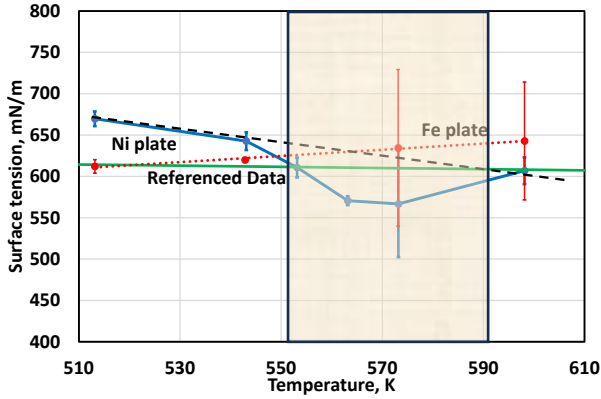


Fig. 5. Surface tension measurements made by: iron plate (red dotted line); nickel plate (blue data) and comparison with data from Ref. [10], (green line); Overall trend of measured data by Ni plate (dashed black line)

This is due to the partial wetting of iron, that indicates the initial coating was not successful. This can be seen on the pictures taken before and after dipping for eight hours. In the case of nickel plate, a complete wetting can be seen in Fig. 7 b), while for iron plate only partial (see upper left corner of the iron plate in Fig. 8 b). Measurements with nickel plate show tendency similar to that in Ref. [10], but instead of linear trend (black dashed line in Fig. 5), the data from our measurements show a valley within the temperature range 550-590 K, (blue curve). This behavior is a result of $\text{SnO} \rightarrow \text{Sn}_3\text{O}_4$ transition (see Fig. 1) and since the data in Ref. [10] are taken in lack of oxygen, this difference in can be expected. Therefore, the implementation of our approach allows identification of the phase transitional uncertainties. Measured surface tension evolves for 1100–1200 s and reaches stable levels, where we assume that oxygen/tin reaction is suppressed (possibly, because all liquid tin at the surface already reacted with the oxygen in the air, see section 2.3).

Table 3. Tested material for Wilhelmy plate

No.	Temp. [K]	$\Delta_{\text{Surf Tens}}(\text{Ni})$ [mN/m]	$\Delta_{\text{Surf Tens}}(\text{Fe})$ [mN/m]
1	598.1	169.8	197.3
2	573.2	91.4	59.3
3	563.2	69.4	-
4	553.2	26.0	-
5	543.1	69.5	53.6
6	513.1	79.5	115.3

Table 3 summarizes recorded surface tensions changes from beginning (pure liquid tin) to final stabilized values, denoted correspondingly as $\Delta_{\text{Surf Tens}}(\text{Ni})$ and $\Delta_{\text{Surf Tens}}(\text{Fe})$. The data obtained with iron plate show greater instability compared to the data, obtained with nickel plate. If measurement continues more than 1200 s the plate (nickel) is capable to further record the

appearance of cracks on the already reacted with the atmospheric oxygen surface.

4.2 Time evolution of surface tension

Equation (1) describes the time evolution of surface tension. This equation somehow appears to be a version of Buttler equation, [17], which in general form for a specie i is:

$$X_{i,s} = X_{i,b} \exp \left[(\sigma - \sigma_i) \frac{A_i}{RT} \right] \quad (10)$$

where $X_{i,s}$ and $X_{i,b}$ are correspondingly the mole surface and bulk fractions of component i ; σ is the surface tension of the mixture, while the surface tension of component i is σ_i . The individual molar surface area of component i , can be calculated from the well-known equation with the assumption that the molecules of each component are approximated as spheres, [17]:

$A_i = \pi \left(\frac{3}{4\pi} \right)^{2/3} N^{1/3} V_i^{2/3}$, where N is Avogadro number, and V_i is the bulk molar volume of component i and $\pi \left(\frac{3}{4\pi} \right)^{2/3} N^{1/3} = 1.02 \times 10^8, \text{ mol}^{-1}$. If we now compare equations (1) and (10), we could separate two important relations. First one is:

$$\sigma(t) = (\sigma - \sigma_i) \quad (11)$$

where, at the initial moment, ($t = 0$), $\sigma(0) = \sigma$ but, because no oxygen is adsorbed on the surface, then it will be equal to pure liquid tin surface tension.

The second equation is related to the partial molar surface area:

$$\frac{1}{b(T)} = \frac{A_i}{RT} \quad (12)$$

To compare the results from the calculations, initial conditions can be applied where only liquid tin is present on the liquid surface. In this case A_i can be calculated involving density and liquid bulk partial molar volume [10,17]. Table 4 presents the results from calculations.

Table 4. Comparison of estimated data for A_i and results from measurements

No	Temp. [K]	$b(T)$, N/m	A_i , Ref. [17]	Ratio $A_i/b(T)$
1	598.1	31682.7	67932.9	2.1
2	573.2	38708.9	67816.7	1.8
3	563.2	39656.6	67770.1	1.7
4	553.2	37355.3	67723.6	1.8
5	543.1	38243.5	67677.0	1.8
6	513.1	49314.7	67538.8	1.4

The results in Table 4 indicate the measured data are approximately 1.8 times lower than calculated in Ref. [17]. However, some references consider this approach for calculation of A_i to be not sufficiently accurate and another equation for calculation of partial molar area is

proposed, [18]: $A_i = 1.02 \times 10^8 V_c^{3/5} V_i^{2/5}$, where V_c is the critical molar volume.

4.3 Investigation of solid materials stability

Based on the “Wilhemmy plate” method, series of tests were conducted in order to estimate structural stability of the selected materials. Plates, made of copper, nickel, iron and spring steel were tested. Each plate, dipped in the liquid tin for measurement was used for determination of the surface tension and the surface energies at the air/tin interface. Different plates made of different materials exhibit two types of behaviour, related to their physical properties. Some of them dissolve in the liquid tin, while others maintain their stability.

Two series of measurements were performed – one with duration eight hours and other with duration five hours at temperature 573 K. The amount of the eroded material from the plate was estimated by image analysis and comparing the surface area before and after dipping. Evaluation of solution rate constant can be performed, based on Eq. 7 as follows. [14]:

$$K = \frac{V}{St} \ln \frac{C_s}{C_s - C} \quad (13)$$

where the density of the melt in case of plate dissolution should be calculated as: $\rho_{melt} = \rho_{Sn} C_{Sn} + \rho_{sol} C_{sol}$. There ρ_{Sn} is the density of the melt (tin) with its fractional concentration C_{Sn} , while ρ_{sol} is the density of dissolved material with its fractional concentration, C_{sol} . The copper plate is severely dissolved in the liquid tin. Comparison between the plate before and after the experiment is shown in Fig. 6. in the case of eight hours dipping time. Based on the images, dissolved solid copper was estimated to be 0.526 g. From Eq. (13) we calculated the solution rate constant for 573 K to be 0.0085 mm/s. In the calculation, data from Table 1 was used as well Refs. [17,18].

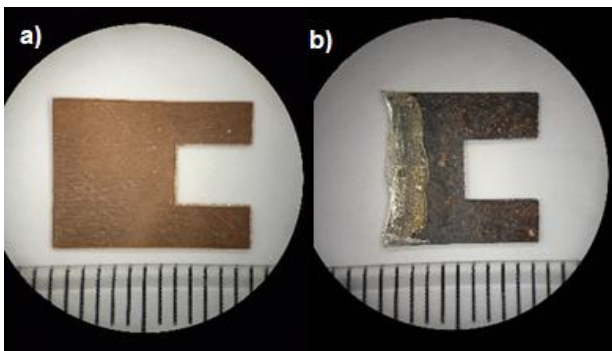


Fig. 6. Comparison between copper plates before (a) and after dipping (b) in molten tin for eight hours at 573 K (the scale is in millimeters)

Surface of the copper plate is covered with oxide after the dipping procedure (the measurement is carried out at the ambient atmosphere). The same procedure is conducted for nickel plate, Fig. 7. In contrast to copper, nickel plate is only slightly eroded after dipping for eight

hours, showing more structural stability against the molten tin.

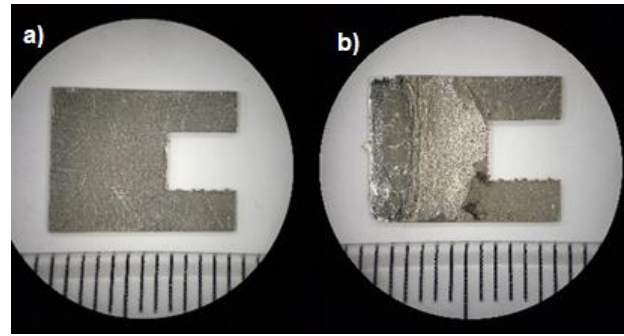


Fig. 7. Comparison between nickel plate before (a) and after dipping (b) in molten tin for eight hours at 573 K (the scale is in millimeters)

A plate made of pure iron was also tested and the image analyse does not show significant erosion (Fig. 8).

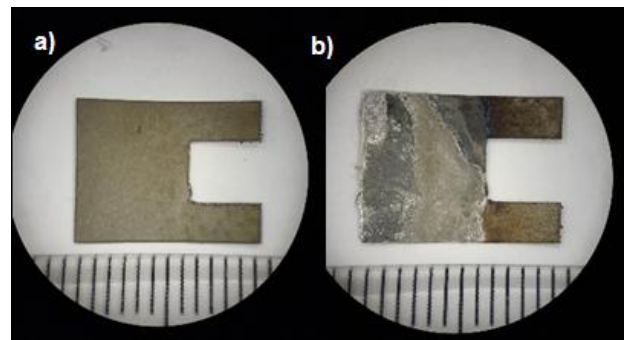


Fig. 8. Comparison between iron plate before (a) and after dipping (b) in molten tin for eight hours at 573 K (the scale is in millimeters)

After the experiments, the iron plate was covered with oxide due to its interaction with the atmosphere but it has excellent structural stability against molten tin in the investigated temperature range. However, the results for surface tension measurement indicate that the iron can not be used as a measuring plate, because of the poor adhesion and not good wetting of the plate, especially in a long-term experiment.

Spring steel plate was also tested and the results are similar to these for pure iron (Fig. 9) i.e., no significant erosion is observed.

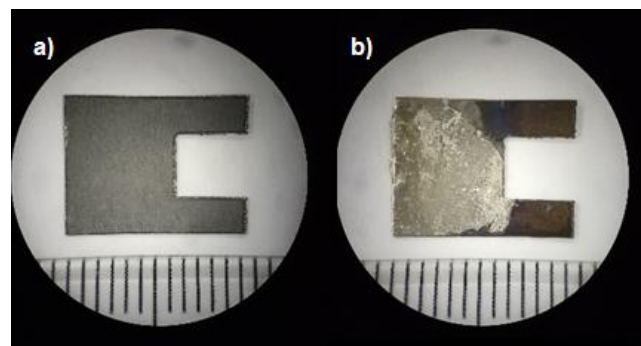


Fig. 9. Comparison between spring steel plate before (a) and after dipping (b) in molten tin for eight hours at 573 K C (the scale is in millimeters)

3.2 Conclusions

The following conclusions are drawn, based on the content, presented in current paper:

- Measurement of tin surface tension, made in transitional $\text{SnO} \rightarrow \text{Sn}_3\text{O}_4$ region, could allow in the future investigation of the phase transitional uncertainties.
- The time evolution of surface tension can be described by Buttler's equation for adsorbed specie. Comparison between measured and calculated initial conditions (the constants A_i and $b(T)$) shows systematic deviation of approximately 1.8 times.
- Structural stability analysis of the selected solid materials indicates that iron, spring steel and nickel do not dissolve in liquid tin for the specified time and temperature intervals. Further investigations are required for higher temperatures and longer periods of time to confirm the long-term structural stability. However, the copper plates dissolved much faster and its solution rate constant was determined.

Further investigations will be directed toward air humidity influence of the surface properties of liquid tin as well as increasing of temperature range for measurements.

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Determination of the Parameter Color of Fresh Engine Oils by Color Indices and Predictive Models

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Abstract: Color prediction tools for motor oils, which can be used to assess their quality are presented in the paper. Three measuring devices working on optical principle for color prediction was used and compared. Color models and color index are defined by RGB, Lab and LCh color components. It was found that the combined use of seven color indices leads to an increase in accuracy for predicting the value characteristics of motor oils.. The obtained results demonstrate that the algorithm used for the analysis of color characteristics of motor lubricating oils allowed the creation of adequate regression models. The proposed research tools can be used in practice, which enables a quick assessment of the quality of motor oils based on the parameter color.

1. Introduction

The quality of the fresh oils is crucial for the excellent performance of both the equipment and the oil itself. Minimal contamination such as mechanical impurities and moisture can be detrimental to providing a stable oil film, heat dissipation, air, and corrosion protection.

The color is one of the main characteristics of motor oils, which gives information about the condition of both fresh and used oils. When moisture gets into the oil, oil color is changed and it becomes cloudy, when soot gets into the oil, it turns black. Upon oxidation, the oil changes color to reddish brown. The test is conducted in laboratory conditions and takes time. It would be much easier if more accessible, mobile devices were used to perform this analysis, which would provide information about the quality of the product. At the current level of development of science and technology, contact and non-contact measuring devices are being developed [1]. Technical instruments working on an optical principle, such as document cameras, digital cameras, colorimeters, video sensors of mobile phones and tablets, have the potential to be used for express analysis of the characteristics of motor oils. Video sensors have the advantage of being affordable, widely distributed and through the data obtained from them models can be created describing the relationship between the characteristics of the analysed product and color components and indices.

The main stage in the development of recognition systems operating on an optical principle is the assessment of the separable motor oils from each other through various features that describe them [2]. It is also possible to predict various characteristics of oils that are usually determined in laboratory conditions and require specific hardware and trained personnel to use it [3]. Based on the research [4] of two viscosity classes of motor oils, it was found that the combined use of color indices leads to an increase in the accuracy of prediction of motor oil characteristic values. It has been shown that seven color indices can be used to predict these characteristics. Analytical models have been created for automated

prediction of the main characteristics of motor lubricating oils. The obtained data can be used to predict a change in their properties, in laboratory conditions.

The main purpose of the present work is to deepen the research by covering all viscosity grades of motor oils available on the market and to determine their quality through color index analysis techniques.

2. Material and methods

The color of the analysed 112 samples of fresh engine oils was determined according to ASTM D 1500. Four viscosity grades were tested, designated as xW-20, xW-30, xW-40 and xW-50 where “x” is from 0 to 20. Measurements were made in a licensed laboratory. They were conducted at room temperature 22 ± 3 °C and relative air humidity 50 ± 5 %RH. The samples are from different batches of motor oils with different applications and operating levels, according to The European Automobile Manufacturers' Association (ACEA). Table 1 shows the mean values and standard deviation of the parameter color measured on fourth viscosity grades motor oils.

Table 1. The characteristics of the tested engine oils

Parameter	xW-20	xW-30	xW-40	xW-50
Color	L2.5-L3.5	L2.5-L5.0	L2.0-L4.0	L3-L6

Table 2 shows the mean values and standard deviation of the color of the engine oils. They are divided by color classes. They are presented with their mean and standard deviation. The data presented is color from the Lab color model.

Table 2. The characteristics of the tested engine oils by color classes

Color	L	a	b
L0	67,66±0,86	-2,65±0,26	-1,8±0,12
L0.5	64,32±2,03	-3,43±1,18	1,85±2,49

Color	L	a	b
L1.0	64,42±1,44	-3,52±0,53	1,22±1,44
L1.5	69,64±2,69	-6,53±1,95	39,99±13,88
L2.0	66,46±4,75	-3,83±2,44	47,95±12,35
L2.5	60,43±6,81	-2,09±2,92	47,21±10,82
L3.0	57,88±12,39	0,33±3,57	50,25±12,64
L3.5	58,96±4,56	2,6±4,74	56,93±7,76
L4.0	47,37±17,09	14,76±22,35	52,08±12,84
L5.0	52,94±2,94	26,41±0,7	60,47±2,45
L5.5	41,92±8,6	32,9±2,77	41,95±7,39
L6.0	45,71±7,11	33,37±2,43	47,25±7,36

To determine the optical characteristics of motor oils, two video sensors and a colorimeter were used, and the examined sample of each oil was 5 ml.

Document camera Triumph Board A 405 (TRIUMPH BOARD a.s., Prague, The Czech Republic). Measurements with this camera's video sensor are in daylight, labelled Dev1. The measurement was made using a 45x40x35 cm styrofoam box. This measurement is labelled Dev2. The camera is placed vertically in the centre of the box and above the sample holder. The distance between the camera and the sample holder is 20 cm. When measuring with video sensors, a region of interest (ROI) measuring 1x1 cm is selected from the resulting color digital image. Colorimeter (Dev3), model PCE-RGB-2 (PCE Holding GmbH, Germany). Manufactured according to standard DIN 5033. The device measures 10-bit RGB color. When measuring with a colorimeter, the color was determined at three points of the sample and their average value was taken. Table 3 summarizes the data on the used measuring devices.

Table 3. Used measuring devices

Designation	Device	Measurement conditions
Dev1	Document camera Triumph Board A 405	Day light
Dev2	Document camera Triumph Board A 405	Homogeneous light
Dev3	Colorimeter PCE-RGB-2	Device light source

2.1 Color models and color indexes

The total number of color features used in the work are 25. 8 color components from the RGB, Lab and LCh color models are defined. Also calculated 6 color indices from the RGB model. 11 indices are calculated from the Lab color model.

The output images of motor lubricating oils of the four classes are obtained in the RGB color model of light. The RGB model is based on three color: red, with a wavelength of 700nm; green, with a wavelength of 546nm; blue, with a wavelength of 436nm.

The 8-bit values of the color components measured by video sensors from the RGB color model are converted in Lab, in the ranges L=[0, 100]; "a" and b=[-100, 100]. Conversion is to CIE L*a*b* 1976. D65 illuminance calculation functions used. Under the same conversion conditions, the color components from the CIE L*C*h*

1976 model were calculated. Since L(Lab)=L(LCh), only the L(Lab) color component was used in the work. Color indices are preferred over the direct use of color components because, according to [5], indices are more effective than color components in separating object classes. Help distinguish the subject from the background in images. Also, the color indices can easily distinguish the levels of white, yellow, or brown between the different analysed objects. The 10-bit values of the RGB color components obtained by the colorimeter are converted to 8-bit, according to the formula:

$$RGB_8 = \frac{RGB_{10}}{1023} \cdot 255 \quad (1)$$

From RGB model are calculated color index using the kind [6]:

$$ri_1 = \frac{2G - R - B}{G + R + B} \quad (2)$$

$$ri_2 = \frac{2G + R - B}{G - R} \quad (3)$$

$$ri_3 = \frac{G + R}{G^2 - RB} \quad (4)$$

$$ri_4 = \frac{G^2 + RB}{G - R} \quad (5)$$

$$ri_5 = \frac{G + R - B}{2G - R - B} \quad (6)$$

$$ri_6 = 2G - R - B \quad (7)$$

where R, G and B are color components from RGB model. The Chroma (C) and hue (h) values from LCh color model are determined by:

$$C = \sqrt{a^2 + b^2} \quad h = \text{atan}\left(\frac{b}{a}\right) \quad (8)$$

Since "h" is obtained in radians it is necessary to convert it to degrees. This is done using the following formula:

$$h^\circ = h_{rad} \frac{180}{\pi} \quad (9)$$

These values were used in the calculation of color indices. The indices are determined by the formulas summarized by Pathare et al. [7].

$$ci_1 = \frac{142,86b}{L} \quad (10)$$

$$ci_2 = 100 - \sqrt{(100 - L)^2 + a^2 + b^2} \quad (11)$$

$$ci_3 = \frac{x-0,31}{0,17} \quad x = \frac{a+1,75L}{5,645L+a-0,012b} \quad (12)$$

$$ci_4 = \sqrt{a^2 + b^2} \quad (13)$$

$$ci_5 = \frac{180 - h}{L + C} \quad (14)$$

$$ci_6 = \frac{2000a}{LC} \quad (15)$$

$$ci_7 = \frac{a}{b} \quad (16)$$

$$ci_8 = \frac{a}{b} + \frac{a}{L} \quad (17)$$

$$ci_9 = L - b \quad (18)$$

$$ci_{10} = \frac{L}{b} \quad (19)$$

$$ci_{11} = \frac{1000a}{L+h} \quad (20)$$

2.2 Methods used for the selection of informative color features.

A more commonly used method for this purpose is the correlation coefficient between the features [8]. This method also has disadvantages related to the type and volume of data used [9]. For this reason, the methods of successively improving estimates can be used, which significantly reduce the number of features. These methods are suitable for feature selection, both for classification and regression of motor oil characteristics such as viscosity, alkalinity, color. The following methods [10], were used to select characteristics describing the motor lubricating oils: SFCPP, FSNCA and FSNCA. The predictive ability of the obtained feature vectors was checked with the PCR and PLSR methods [2]. A second-order polynomial model [11] was used, through which the main characteristics of motor oils were predicted. Coefficient of determination, Fisher's test and Student's test were used to evaluate this model.

3. Results and discussion

3.1 Selection of informative color features and indices for color prediction (L).

Figure 1 shows a graph of the results of a selection of informative color features for color prediction. Data is for Dev1. The signs are plotted on the horizontal axis, and the values of their weighting coefficients on the vertical axis.

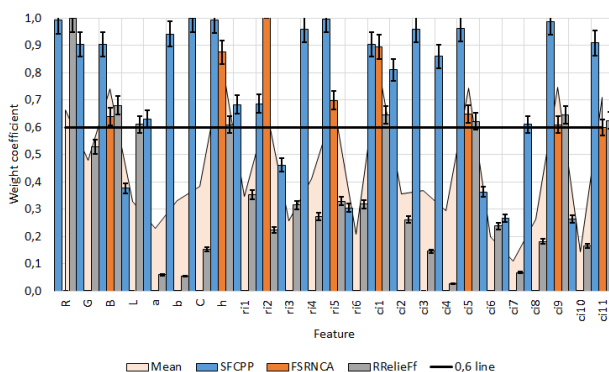


Fig. 1. Feature selection for regression on data from Dev1

Table 4 plots the resulting feature vectors for Dev1. The most traits, compared to the other selection methods, were selected by the SFCPP method – a total of 18 traits. All components from the RGB model and two components each from Lab and LCh are selected. Only two are the RGB indices, while the Lab indices are eight. In the FSRNCA method, one color component is selected from the RGB and LCh models. Two are the RGB indices and four are the Lab indices. According to the RReliefF method, two components from the RGB model and one

each from Lab and LCh are selected. There are no RGB indices selected, while the Lab indices are four.

Table 4. Feature selection for regression on data from Dev1

Vector of features	Selection method	Feature selection
CFV1D1	SFCPP	R G B a b C h ri2 ri4 ri5 ci1 ci2 ci3 ci4 ci5 ci8 ci9 ci11
CFV2D1	FSRNCA	B h ri2 ri5 ci1 ci5 ci9 ci11
CFV3D1	RReliefF	R B L h ci1 ci5 ci9 ci11

Figure 2 shows a graph of the results of a selection of informative color features for color prediction. Data is for Dev2

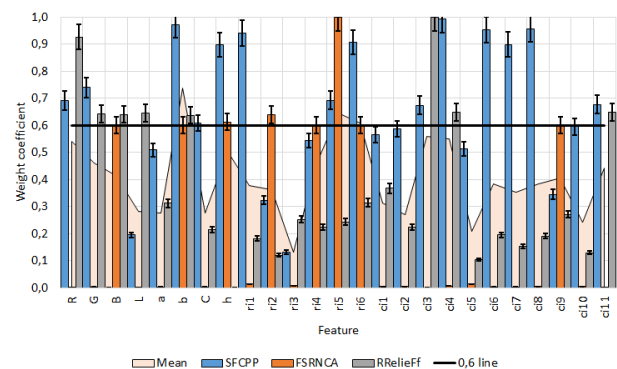


Fig. 2. Feature selection for regression on data from Dev2

Table 5 plots the resulting feature vectors for Dev2. The SFCPP method has the most selected features compared to the other methods. Two components from RGB are selected and one each from Lab and LCh. Only 3 are the RGB indices, while the Lab indices are seven. According to the FSRNCA method, one component each from RGB, Lab and LCh was selected. There are four RGB indexes and only one Lab index. All three components from the RGB model and two from the Lab were selected using the RReliefF method. There are no RGB indices selected, and there are three Lab indices.

Table 5. Selected features based on data from Dev2

Vector of features	Selection method	Feature selection
CFV1D2	SFCPP	R G b h ri1 ri5 ri6 ci3 ci4 ci6 ci7 ci8 ci10 ci11
CFV2D2	FSRNCA	B b h ri2 ri4 ri5 ri6 ci9
CFV3D2	RReliefF	R G B L b ci3 ci4 ci11

Figure 3 shows a graph of the results of a selection of informative color features for color prediction. Data is for Dev 3.

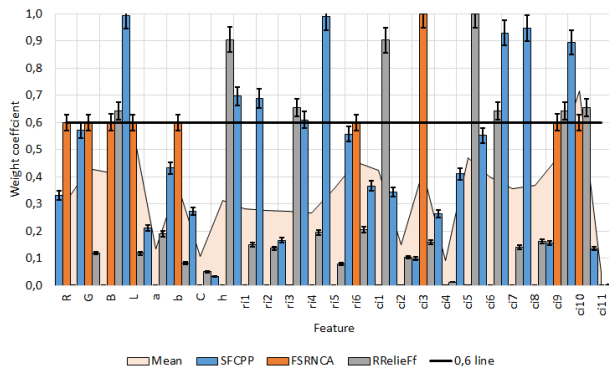


Fig. 3. Feature selection for regression on data from Dev3

Table 6 plots the obtained feature vectors for Dev3. Using the SFCPP method, one color component was selected, as well as three RGB and Lab indices each. In the FSRNCA method, all three color components from the RGB model and two from Lab are selected. Only one RGB index is selected, while the Lab indices are three. According to the RReliefF method, one color component from RGB and LCh is selected each. There is only one RGB index selected, while the Lab indices are five.

Table 6. Selected features based on data from Dev3

Vector of features	Selection method	Feature selection
CFV1D3	SFCPP	L ri1 ri2 ri4 ci7 ci8 ci10
CFV2D3	FSRNCA	R G B L b ri5 ci3 ci9 ci10
CFV3D3	RReliefF	B h ri3 ci1 ci5 ci6 ci9 ci10

3.2 Verification of the ability to predict the color of fresh motor oils by selected feature vectors.

The ability to predict the color of fresh motor oils was evaluated. Partial least squares regression (PLSR) and principal component regression (PCR) methods were applied. Data from the selected feature vectors were used. These vectors were reduced to the number of components (LVs and PCs) that described more than 95% of the variance in the data. Mean squared error (SSE) and root mean squared error (RMSE) were estimated. The coefficient of determination (R^2) was also used as a criterion. The assessment of predictability is based on the accuracy of a linear model obtained between the principal components or latent variables to which the feature vectors have been reduced and the corresponding motor oil characteristic to be predicted. The results of testing the predictive ability of the selected feature vectors on data from Dev1 are shown in Table 7. When using PLSR to predict the color of motor oils, coefficient of determination values (up to 0,38) were observed, which were slightly higher than those obtained by PCR (reaching 0,24-0,38). On the other hand, the errors of PLSR are slightly larger than those of PCR. In PLSR, the SSE errors reach values as high as 2,22, and in PCR as high as 2,2. The RMSE errors are the same for both regression methods and reach a maximum value of 0,53.

Table 7. PLSR and PCR results for color prediction on Dev 1 data

Vector of features	PLSR				PCR			
	SS E	R^2	RMS E	LV s	SS E	R^2	RMS E	PC s
CFV1 D1	2,19	0,37	0,52	3	2,14	0,35	0,52	3
CFV2 D1	2,22	0,38	0,53	2	2,22	0,38	0,53	2
CFV3 D1	2,2	0,37	0,52	2	1,7	0,24	0,46	2

The results of testing the predictive ability of the selected feature vectors on Dev2 data are shown in Table 8. When using PLSR to predict the color of motor oils, coefficient of determination values (up to 0,43) were observed that were slightly higher than those obtained by PCR (reaching 0,32-0,42). On the other hand, the errors of PLSR are slightly larger than those of PCR. In PLSR, the SSE errors reach values as high as 2,31, and in PCR as high as 2,3. The RMSE errors are the same for both regression methods and reach a maximum value of 0,54.

Table 8. PLSR and PCR results for color prediction on Dev 2 data

Vector of features	PLSR				PCR			
	SS E	R^2	RMS E	LV s	SS E	R^2	RMS E	PC s
CFV1 D2	2,03	0,32	0,51	3	2,04	0,32	0,51	3
CFV2 D2	2,28	0,41	0,53	2	2,28	0,41	0,53	2
CFV3 D2	2,31	0,43	0,54	2	2,3	0,42	0,54	2

The results of testing the predictive ability of the selected feature vectors on Dev3 data are shown in Table 9. When using PLSR to predict the color of motor oils, coefficient of determination values (up to 0,24) were observed that were slightly larger than those obtained by PCR (reaching 0,19-0,24). On the other hand, the errors of PLSR are slightly larger than those of PCR. In PLSR, SSE errors reach values up to 16,93, and in PCR up to 12-17. The RMSE errors are the same for both regression methods and reach a maximum value of 0,46.

Table 9. PLSR and PCR results for color prediction on Dev 3 data

Vector of features	PLSR				PCR			
	SSE	R^2	RMSE	LV s	SSE	R^2	RMSE	PCs
CFV1 D3	14,46	0,19	0,43	2	14,42	0,19	0,43	2
CFV2 D3	11,68	0,15	0,38	2	11,68	0,15	0,38	2
CFV3 D3	16,93	0,24	0,46	1	16,93	0,24	0,46	1

The obtained results show that the models obtained from data from a document camera when shooting with homogeneous light (Dev2) and those obtained with a colorimeter (Dev3) describe the experimental data with sufficient accuracy. Images captured in ambient light and the resulting color indices are greatly affected by ambient noise. This is proven by the obtained predictive models and the insufficient accuracy with which the experimental data are described and their poor predictive power. Models obtained with a document camera under homogeneous lighting (Dev2) are about 5% more accurate than those obtained with the same device but under daylight (Dev1). Among the compared devices, the colorimeter (Dev3) shows the highest accuracy and repeatability of the obtained results. Additional analysis was done on the patterns obtained from data from this device. Figure 4 shows a general type of model obtained from Dev3 colorimeter data. The independent variables x and y plotted on the horizontal axis represent the first and second principal components. The dependent variable z is a predicted characteristic of motor oils. The analysis of the residuals of this model showed that they have a normal distribution and are closely located on the normal probability plot.

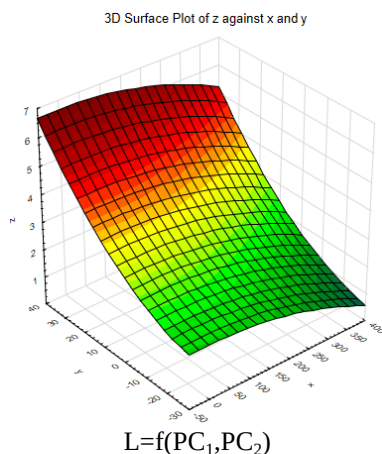


Fig. 4. Model obtained by data from Dev 3 – general view

3.3 Establishment of prediction model for color of engine oils.

From the preliminary analyses made on the selection of informative features and the assessment of the predictive ability of the vectors composed of them, it was found that after reducing the vectors of features with principal components, lower values of the coefficient of determination are obtained, compared to the reduction by latent variables with PLSR method. On the other hand, the principal component errors are lower. This is a prerequisite after reducing the feature vectors to obtain regression predictive models for the main properties of motor oils that describe the experimental data with sufficient accuracy. For this reason, feature vectors reduced with principal components were used to create regression models. Also, preliminary analysis found that the data obtained with Dev3 lacked sufficient informativeness. This is probably due to the specifics of

the operation of this device. For this reason, no regression models were built on Dev3 data.

Establishment of regression models based on data for Dev 1. Based on data from Dev1, feature vectors that have the greatest predictive ability for the main characteristics of motor oils have been selected. Selected feature vectors are summarized in Table 10

Table 10. Vectors and their corresponding features on data from Dev 1

Vector	Features
VFV3D1	B C ri2 ri3 ri4 ri5 ci5 ci6 ci10 ci11
CFV2D1	B h ri2 ri5 ci1 ci5 ci9 ci11
TFV2D1	R B L h ri2 ri5 ci9

Data from the CFV2D1 trait vector were reduced to two principal components. After removing the non-significant coefficients from the main model that had a $p\text{-Value} \gg \alpha$, a model with three coefficients was obtained. A model for predicting L of the form was obtained:

$$L = 7,16 - 0,01PC_1 + 0,01PC_2 - 0,001PC_1^2 - 0,00013PC_1PC_2 \quad (21)$$

The resulting model was evaluated. Table 11 shows the results of this check. A low standard error value is observed. The coefficients of the model are significant because the $p\text{-Value}$ is much less than the accepted significance level $\alpha=0,05$. The error value SE is low. According to Fisher's criterion, the calculated F is much lower than its critical stability under the corresponding degrees of freedom. The t values of the model coefficients are also much higher than the critical one.

The coefficient of determination has a value above 0,6. The results show that the obtained model describes a significant part of the variation of the kinetic viscosity of motor oils. These parameters are not a sufficient criterion to evaluate the model. It is necessary to analyse the residuals.

Table 11. Data for estimating the resulting regression model for L on data from Dev1

$R^2=0,63$ $F(4,77)=12,699$ $p<0,00$ Std. Error of estimate: 0,86 $F_{cr}(4,77)=2,49$ $t_{cr}(77)=1,67$						
N=82	b*	Std.Err. of b*	b	Std.Err. of b	t(77)	P-value
Intercept			7,16	0,59	12,049	0,00
x	-0,64	0,15	-0,01	0,002	-4,23	0,00
y	0,26	0,1	0,01	0,006	2,59	0,01
x^2	-1,1	0,16	-0,001	0,0001	-6,77	0,00
$x*y$	-0,2	0,1	-0,00013	0,0001	-1,99	0,05

The resulting model and plots of its residuals are shown in Figure 5. The first two principal components, which are the independent variables in the model, are

plotted on the horizontal axes. On the vertical axis are those of the dependent predicted variable. You can see the area of variation of the two factors where the dependent variable has the largest values is when they are at their upper levels. A sign of the normal distribution of the residuals is their location on a straight line. The distribution of the residuals and their location around the normal line, in the normal probability plot, they are close to a normal distribution. When analysing the residuals, it is established that there is no systematic deviation of the actual data from the theoretical ones, which is also a sign of their normal distribution.

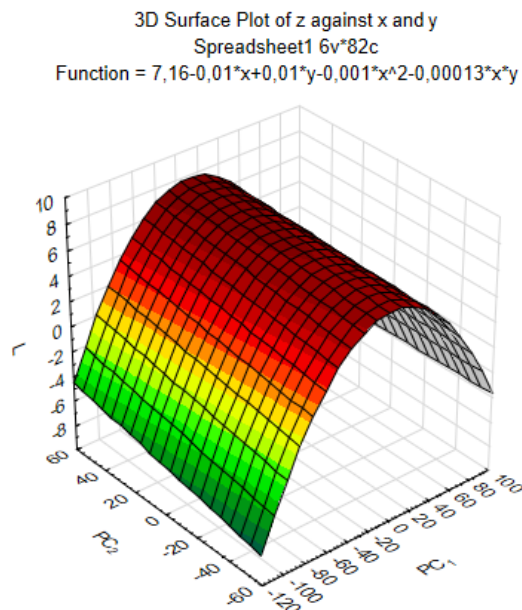


Fig. 5.a) general view

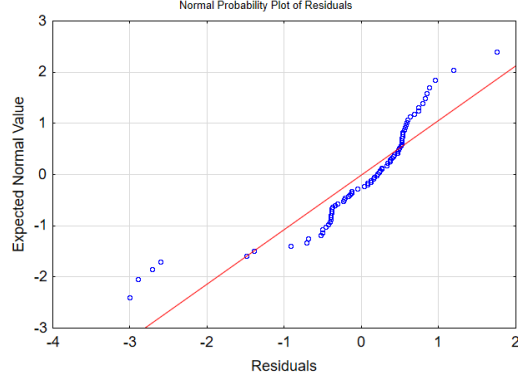


Fig. 5.b) a normal probability plot of residuals value

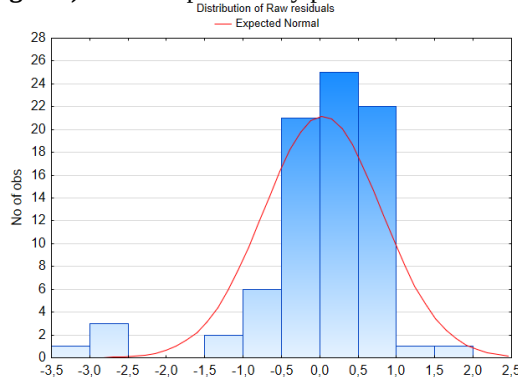


Fig. 5.c) residual values distribution

Fig. 5. A model of the type $L=f(PC_1, PC_2)$ by data from CFV2D1 from Dev1

3.4 Establishment of regression models based on data for Dev 2.

Based on data from Dev2, feature vectors have been selected that have the greatest predictive ability for the main characteristics of motor oils. Selected feature vectors are summarized in Table 12.

Table 12. Vectors and their corresponding features on data from Dev 2

Vector	Features
VFV3D2	B L ri4 ri5 ci3 ci4 ci11
CFV3D2	R G B L b ci3 ci4 ci11
TFV3D2	a b ri1 ri2 ri3 ri4 ri5 ri6 ci1 ci2 ci4 ci11

Creating a regression model for color. Data from the CFV3D2 character vector were reduced to two principal components. After removing the non-significant coefficients from the main model that had a $p\text{-Value} \gg \alpha$, a model with three coefficients was obtained. A model for predicting L of the form was obtained:

$$L = 3,35 + 1,7 \cdot 10^{-4} PC_1 + 0,012 PC_2 - 1,6 \cdot 10^{-5} PC_2^2 \quad (22)$$

Table 13 shows data for evaluating the resulting model. The resulting model was evaluated. A low standard error value is observed.

The coefficients of the model are significant because the $p\text{-Value}$ is much less than the accepted significance level $\alpha=0.05$. The error value SE is low.

According to Fisher's criterion, the calculated F is much lower than its critical stability under the corresponding degrees of freedom. The t values of the model coefficients are also much higher than the critical one.

The coefficient of determination has a value above 0,8. The results show that the obtained model describes a significant part of the variation of the kinetic viscosity of motor oils.

These parameters are not a sufficient criterion to evaluate the model. It is necessary to analyse the residuals.

Table 13. Data for estimating the resulting regression model for L on data from Dev2

$R^2=0,81$ $F(3,78)=49,93$ $p<0,00$ Std.Error of estimate: 0,64 $F_{cr}(3,78)=2,72$ $t_{cr}(78)=1,67$						
N=82	b*	Std.Err. of b*	b	Std.Err. of b	t(78)	p-value
Intercept			3,35	0,086	39,23	0,00
x	0,055	0,067	1,7E-4	0,00	0,82	0,04
y	1,133	0,122	0,012	0,00	9,31	0,00
y ²	0,415	0,121	1,6E-5	0,00	-3,43	0,00

The resulting model and plots of its residuals are shown in Figure 6. The first two principal components, which are

the independent variables in the model, are plotted on the horizontal axes. On the vertical axis are those of the dependent predicted variable. You can see the area of variation of the two factors where the dependent variable has the largest values is when they are at their upper levels. A sign of the normal distribution of the residuals is their location on a straight line. The distribution of the residuals and their location around the normal line, in the normal probability plot, they are close to a normal distribution. When analysing the residuals, it is established that there is no systematic deviation of the actual data from the theoretical ones, which is also a sign of their normal distribution. From here it can be considered that the prerequisites of the regression analysis are fulfilled.

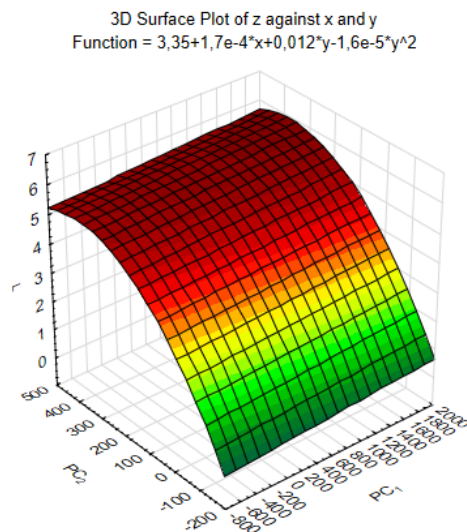


Fig. 6.a) general view

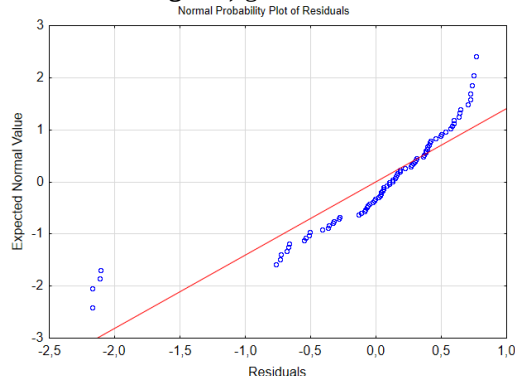


Fig. 6.b) a normal probability plot of residuals value

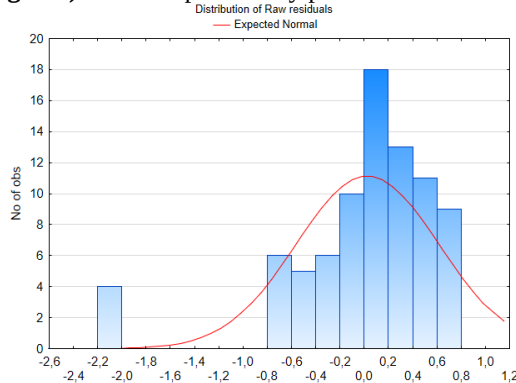


Fig. 6.c) residual values distribution

Fig. 6. A model of the type $L=f(PC_1, PC_2)$ by data from CFV2D1 from Dev 2

3.5. Comparative analysis of the obtained results

The research in the present work is related to the assessment of the possibility of predicting the main characteristics of motor lubricating oils through the analysis of their color signs and indices. Informative features have been established for these products using color digital images and colorimeter data. In Table 14, a comparative analysis of the used methods is made and the results of their use in the present study is presented. During the selection of features, it was found that they have different predictive capabilities with respect to the investigated characteristics of motor oils, which is confirmed by the research done using partial least squares regression and principal component regression. This is also confirmed by the analysis of the created predictive models, describing with sufficient accuracy the relationship between the selected features and the characteristics of motor oils.

Table 14. A comparative analyses of the used methods

Research type	Color (L)
Informative features selection	Color components and Lab indices are mainly selected. In a lesser degree RGB indices
Predictability assessment	Prediction accuracy depends on the device used. Dev1, $R^2=0,37-0,38$; Dev2, $R^2=0,32-0,41$; Dev3, $R^2=0,19-0,24$
Predictive models creation	Prediction accuracy depends on used device: Dev1, $R^2=0,65$; Dev2, $R^2=0,81$

4 Conclusion

The methods and algorithms adapted and applied in the present work for tracking the change of main characteristics of motor lubricating oils, depending on their color characteristics, are entirely aimed at using methods that would be sufficiently effective in terms of quick and simplified classification and at the same time giving satisfactory accuracy according to the technological requirements for the characteristics of motor oils.

According to the obtained results, the FSRNCA and RReliefF methods were found to be suitable for the selection of color features for creating regression models.

Tools have been developed to predict the main characteristics of motor oils, which is based on the main color components and color indices calculated from them and on a certain set of ratios between them.

A comparative study was made to evaluate the influence of the used technical measuring tools on the accuracy of forecasting the main characteristics of motor lubricating oils.

It was found that the use of a video camera and homogeneous lighting gives better results in predicting the specified characteristics than a video camera used in daylight, as well as a colorimeter working with the light reflected from the object.

It has been found that the individual color components, in combination with Lab indices, are suitable for predicting the color of oils. Using color components to predict this characteristic of motor oils is inappropriate.

The obtained results demonstrate that the algorithm used for the analysis of color characteristics of motor lubricating oils allowed the creation of adequate regression models.

These models allow quick and non-destructive determination of main characteristics of motor oils, in the specified measurement conditions. As confirmed by the results, the obtained predictive models manage to keep low values of errors and high values of the coefficient of determination, especially in the class corresponding to the total alkalinity of the oils, which is used as a criterion by manufacturers for the sale of lubricant products. This fact has the prospect of directly applying data from visual images, in combination with selected predictive models, in systems for evaluating the quality of motor lubricating oils.

Acknowledgements

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Design, research and production of gear wheels for mining machines

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Abstract. The current state of the Ukrainian fuel and energy complex requires a continuous increase in coal production. As part of concept implementation for the coal industry development now created and introduced into serial production a new generation of mining machines. The most loaded element of mining machine is its gear wheels of cutting part. Calculations for the strength and durability of gearings have been carried out. The modelling of the teeth stress-strain state by the finite element method has also been carried out.

Works were carried out on the processing of parts of gearboxes of mining machines.

Investigations of the processes of blade processing of gears and shafts of gearboxes of coal combines were carried out. The state of the surface layer, the quality and accuracy of processing these parts with a blade tool made of superhard materials are determined. Researches of grinding of an involute profile of gear wheels of reducers of mining machines are carried out. A study was made of the surface quality and accuracy of gears after grinding with special abrasive wheels.

The influence of processing processes with a tool made of superhard materials on the strength and durability of gear drives of mining machines is determined.

1 The actuality of the task

The current state of the Ukrainian fuel and energy complex requires a continuous increase in coal production. Currently, 80% of coal deposits are in thin seams. To solve this problem Ukrainian plants created and introduced into serial production a new generation of coal shearers.

An example of a new design is UKD200-500 coal shearer. It is designed for mechanized coal extraction as part of mining complexes, in longwalls of shallow and inclined seams 0,85–1,5 m thickness, moving along strike with inclination angles up to 35°, as well as rise and fall with angles up to 10°, with coal cutting resistance up to 480 kN/m. Its technical characteristics are at the level of modern foreign counterparts.

UKD200-500 coal shearer is a cutting action machine equipped with auger executive bodies for destruction and loading of coal onto a conveyor – Fig. 1 Moving the shearer with a scraper conveyor is carried out by means of an external feed system.

The most important and loaded element of the shearer is its cutting part – Fig. 2, 3. It is a three-stage spur reducer with an electric motor:

- 1st stage is three in-line cylindrical gears;
- 2nd stage is planetary single-stage with fixed epicycle;
- 3rd stage is four in-line cylindrical gears.



Fig. 1. The UKD200-500 coal shearer.

– Darnel K4-16-233 electric motor, rated power 250 kW, rated speed 1470 rpm, asynchronous three-phase with squirrel-cage rotor.

The cutting part of the UKD200-500 coal shearer is unified. This allows it to be installed on either the left or right side of the frame. Each reducer stage is located in a separate sealed chamber, protected from the penetration of abrasive particles.

The material of external gears is Steel 20Ch2N4ASh, with heat treatment – carburizing followed by quenching and low tempering. Material of the planetary stage epicycle – Steel 40 ChN, with heat treatment – improvement for hardness 270–300HB.

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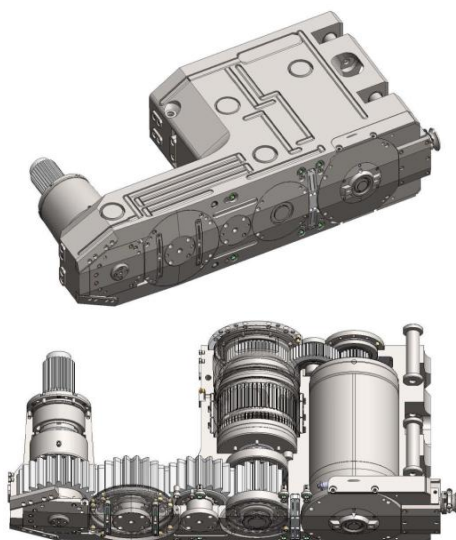


Fig. 2. The cutting part of UKD200-500 coal shearer.

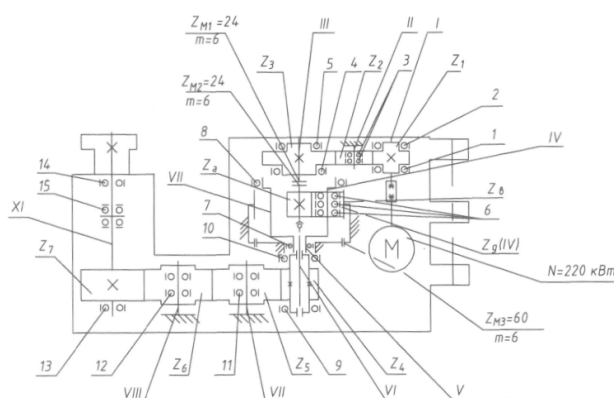


Fig. 3. Kinematic diagram of the cutting part reducer.

2 Problem statement

Reliability and durability of the shearer as a whole depend on the reliability and durability of the cutting part, mainly its gears. It is necessary to take into account that coal mining equipment operates in especially difficult conditions. These are elevated temperatures, a wide range of shock loads and extremely high dust levels. In this case, it is required to ensure the high durability of gears and bearings – at least 15000 hours. Based on this, during the design process, as well as in the future, taking into account the operating experience, calculations for contact and bending durability and static strength of gearings was performed.

The reliability and durability of gears also depend on the accuracy of their manufacture. Therefore, when organizing production, it was necessary to increase the accuracy of the teeth. To do this, we carried out work on the study of the processing of large-module gears for gearboxes of mining machines with the use of modern abrasive and cutting tools.

Compared to other processing methods, this technology allows to significantly increase wear resistance, reduce the coefficient of friction, increase the accuracy of processing and reduce the height parameters of the

roughness of gear wheels and, thereby, increase the reliability and service life of gears.

Next, we will consider the methods, approaches and results of calculations and experimental studies of cutting processing and profile grinding.

3 Materials and methods

3.1 The basic initial data

The design load was taken as follows:

- rated torque $T_r = 1624 \text{ N}\cdot\text{m}$;
- the maximum long-term operating torque for calculating the durability $T_{rM} = 3090 \text{ N}\cdot\text{m}$;
- the maximum short-term torque for calculating the static strength $T_M = 5740 \text{ N}\cdot\text{m}$.

The total estimated resource of the combine is 15000 hours. We assume that each of the two cutting parts works for half of the calculated resource with nominal loads. Therefore, the estimated life for the cutting part is 7500 hours.

3.2 Calculations of the gear's geometry, contact and bending stresses

The calculation of the gearing geometry, contact and bending stresses in them was carried out based on standard techniques [1–4]. Also, the permissible stresses were refined based on mathematical modeling for fatigue processes in the teeth [5].

The assessment of durability was carried out using the recommendations of GOST 21354–87 [4]: the slight slope of the right branch of the contact fatigue curve was taken into account at the total numbers of stress change cycles $N_K > N_{Hlim}$. Here N_{Hlim} is the base numbers of cycles corresponding to the contact fatigue limit $\sigma_{Hlim b}$ – Fig. 4.

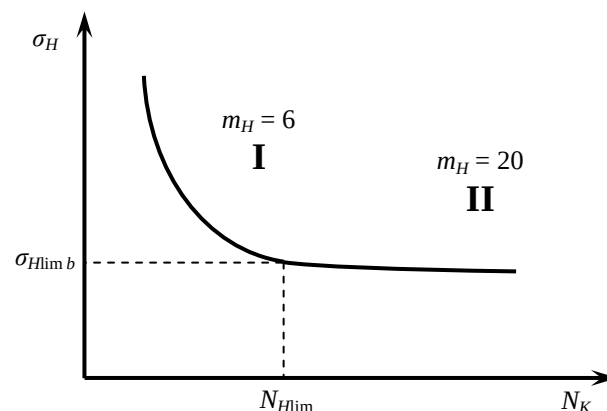


Fig. 4. Contact fatigue curve of active tooth surfaces: zone of limited (I) and long-term fatigue (II).

The resource calculation was carried out in the next sequence.

1. The equivalent numbers of stress change cycles were determined for calculating the contact and bending fatigue N_{HE} and N_{FE} according to the next formulas [6]:

$$N_{HE} = \sum_{i=1}^{i_n} \left(\frac{T_{li}}{T_{1H}} \right)^3 N_{ci};$$

$$N_{FE} = \sum_{i=1}^{i_n} \left(\frac{T_{li}}{T_{1F}} \right)^{q_F} N_{ci} \leq N_{F \lim}, \quad (1)$$

where $i = 1; \dots; i_n$ – accepted for calculation load steps in the cyclorama; T_{li} and $N_{ci} = 60n_i \cdot t_i$ – corresponding values of torque and stress cycles on the pinion; $N_{F \lim}$ – base number of cycles corresponding to the bending fatigue limit $\sigma_{F \lim b}$; $q_F = 9$ for the case of surface chemo-heat treatment of teeth and an unpolished tooth root; $q_F = 6$ for other variants of heat treatment or polished tooth root.

Additionally, for case $N_k > N_{H \lim}$, steps with loads that create stresses below the so-called damaging level $\sigma_{HG} = \alpha_{HG} \cdot \sigma_{H \lim b}$ were excluded from the cyclorama. GOST 21354–87 recommends $\alpha_{HG} = 0,75$ [4].

2. The total number of cycles to failure was calculated.

2.1. Bending fatigue:

$$\left. \begin{aligned} N_{F\Sigma 1,2} &= N_{FE1(2)} \left(\frac{\sigma_{FP1,2}}{\sigma_{F1,2}} \right)^{q_F} & \text{if } N_{F\Sigma 1,2} \leq N_{F \lim 1,2}; \\ N_{F\Sigma 1,2} &= \infty & \text{if } N_{F\Sigma 1,2} > N_{F \lim 1,2}. \end{aligned} \right\} \quad (2)$$

2.2. Contact fatigue:

$$\left. \begin{aligned} N_{H\Sigma} &= N_{HE} \left(\frac{\sigma_{HP}}{\sigma_H} \right)^6 & \text{if } N_{HE} \leq N_{H \lim}; \\ N_{H\Sigma} &= N_{HE} \left(\frac{\sigma_{HP}}{\sigma_H} \right)^{20} & \text{if } N_{HE} > N_{H \lim}. \end{aligned} \right\} \quad (3)$$

3. The calculated durability of the teeth was determined by contact and bending fatigue in hours, L_{Hh} and L_{Fh} :

$$L_{Hh} = t \frac{N_{H\Sigma}}{N_{HE}}; \quad L_{Fh} = \min \left\{ t \frac{N_{F\Sigma 1}}{N_{FE1}}, t \frac{N_{F\Sigma 2}}{N_{FE2}} \right\}, \quad (4)$$

where t – required resource of gears.

The minimum of L_{Hh} and L_{Fh} was taken as the final value of the durability L_h for each gear pair.

Also, modeling of the stress-strain state for gears by the finite element method has been carried out. Finite element analysis was carried out in specialized software ANSYS. Gears, carrier of the planetary stage, reducer housing, shafts, spline parts, etc. were research.

3.3 Experimental research of milling and profile grinding of teeth

The increase in productivity, accuracy and quality of processing of hardened gear wheels of reducers of mining machines is ensured by the improvement of the technology and kinematics of tooth milling and tooth grinding, the development of instrumental and technological equipment for the processing process. The main allowance after heat treatment for tooth grinding up

to 90% should be performed with a blade tool due to high-speed tooth processing with special cutters [7].

The bearing capacity of gears in terms of contact strength increases with an increase in the surface hardness of the teeth. However, the high complexity of manufacturing hardened large-module gears limits their use due to significant allowances.

Milling cutters can be equipped with cutting plates made of hard alloy, ceramics, superhard materials to obtain the necessary accuracy and quality of surface treatment of gear wheels. Due to the intensification of cutting modes when using cubic boron nitride (CBN) plates, productivity increases by 2–5 times without reducing the quality and accuracy of processing. The use of CBN cutters makes it possible to significantly increase the reliability and durability of gear wheels due to the high quality of processing [8].

Tests of the process of profile grinding of gears were carried out on a tooth grinding machine with a CNC mod. "HÖFLER RAPID 1250" – Fig. 5.



a



b

Fig. 5. Grinding wheels made of different abrasive materials (a) and installing the grinding wheel on the machine (b)

During the study of the process, the gear grinding power and all other measurements of the precision parameters of the processed gears were recorded [9].

Gear wheels of the same standard size made of hardened steel with a hardness of up to HRC 60 were ground. The outer diameter of the gear wheel $d = 233.4$ mm; number of teeth $z = 29$; module $m = 7$ mm; tooth helix angle $\beta = -18^\circ$; crown width $b = 60$ mm.

The following highly porous abrasive wheels were tested:

400-32-127 WG946 Hs12 VS – monocorundum;
400-32-127 3SG46Hs 12 VS – zol-gel monocorundum;
400-32-127 A9946Hs 12 VS – white corundum;
400-32-127 A8960K9V – chrome corundum;
400-32-127 25AF46L6V – white corundum with normal porosity.

4 Results and discussion

4.1 Calculation results of gears

Calculation results of strength and durability for gearings of the cutting part reducer are presented in Tables 1–3.

Table 1. Calculation results of gears strength and durability for 1st stage.

Gear	Z_1	Z_2	Z_3
Number of teeth	23	43	50
Module m , mm	6		
Contact fatigue safety factor S_H ($[S_H] = 1.2$)	1.45	1.74	1.90
Bending fatigue safety factor S_F ($[S_F] = 1.55$)	2.72	2.12	2.97
Contact static strength safety factor S_{HM} ($[S_{HM}] = 1$)	1.93	1.93	2.32
Bending static strength safety factor S_{FM} ($[S_{FM}] = 1.75$)	3.62	3.50	3.59
Calculated service life of teeth t , hours	330200	$12.7 \cdot 10^6$	$73.5 \cdot 10^6$

Table 2. Calculation results of gears strength and durability for 2nd (planetary) stage.

Gear	Z_a	Z_R	Z_b
Number of teeth	15	23	60
Module m , mm	6		
Contact fatigue safety factor S_H ($[S_H] = 1.2$)	1.24	1.72	1.42 [S_H]=1.1
Bending fatigue safety factor S_F ($[S_F] = 1.55$)	2.83	2.45	3.19 [S_F]=1.7
Contact static strength safety factor S_{HM} ($[S_{HM}] = 1$)	1.71	1.71	1.69
Bending static strength safety factor S_{FM} ($[S_{FM}] = 1.75$)	3.79	3.69	4.74
Calculated service life of teeth t , hours	14400	810500	$1.2 \cdot 10^6$

Table 3. Calculation results of gears strength and durability for 3d stage.

Gear	Z_4	Z_5	Z_6	Z_7
Number of teeth	14	20	20	24
Module m , mm	16			
Contact fatigue safety factor S_H ($[S_H] = 1.2$)	1.66	1.98	2.19	2.01
Bending fatigue safety factor S_F ($[S_F] = 1.55$)	2.92	2.78	2.78	3.05
Contact static strength safety factor S_{HM} ($[S_{HM}] = 1$)	1.50	1.50	1.65	1.65
Bending static strength safety factor S_{FM} ($[S_{FM}] = 1.75$)	2.89	3.52	3.87	2.77
Calculated service life of teeth t , hours	52500	151300	277100	165600

Analysis of the data in this tables shows that the strength and durability for all gears of the cutting part are ensured.

Also, we will give an example of modeling the stress-strain state for gears using the finite element method. Fig. 6 shows Finite Element model of gear pair – her general view and area in the contact of teeth.

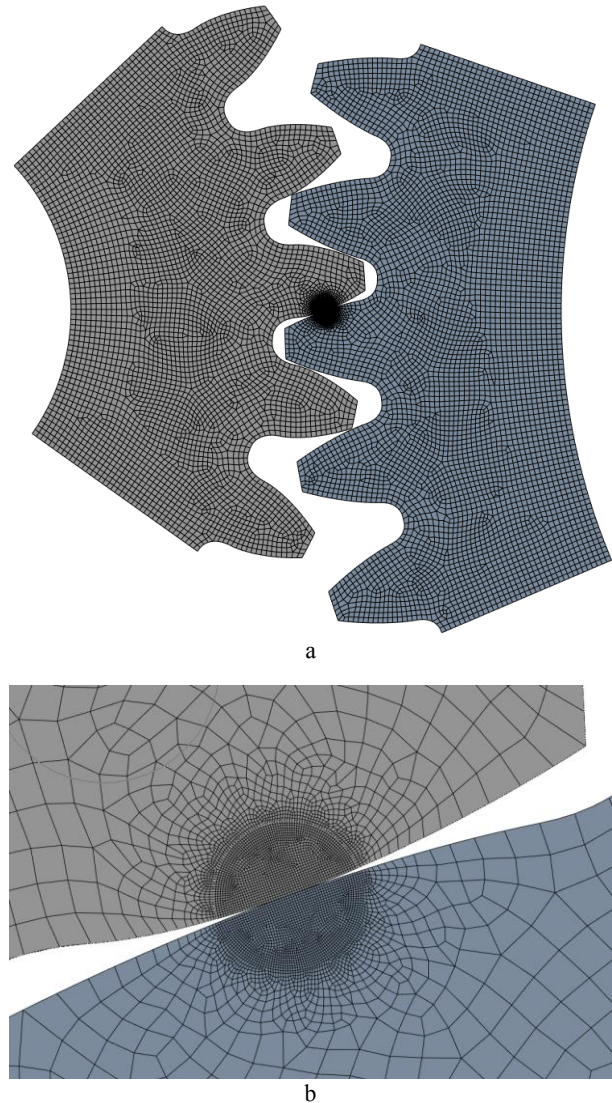


Fig. 6. Finite Element model of gear pair: general view (a) and area at the contact place (b)

Fig. 7 shows the distribution of equivalent stress by von Mises, **MPa**, in contact of first pair of the cutting part reducer. Fig. 8 shows the distribution of bending stress, **MPa**, for pinion of their pair.

4.2 Results of experimental research for profile grinding

The test results of grinding wheels made of monocrystalline and zol-gel corundum showed their high efficiency in comparison with wheels made of white electrocorundum. These wheels provide a reduction in grinding power and better indicators in terms of accuracy and quality of processing.

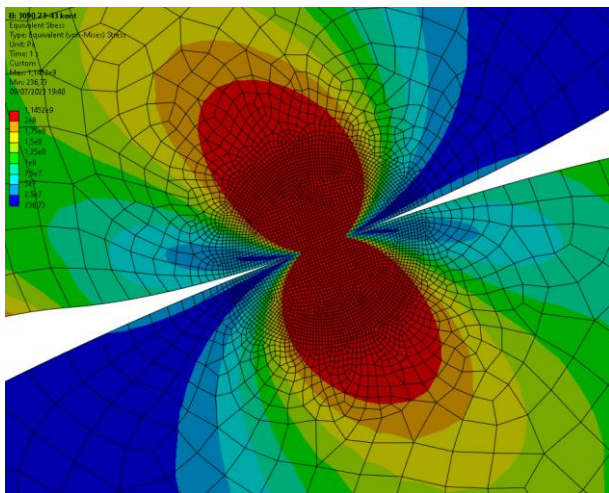


Fig. 7. Equivalent stress by von Mises, MPa, in contact of first pair of the cutting part reducer.

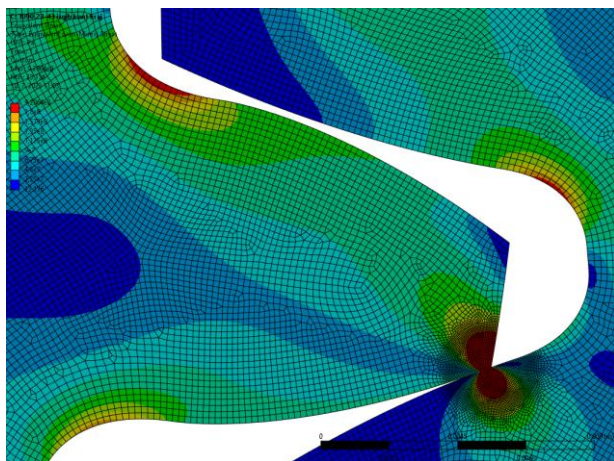


Fig. 8. Bending stress, MPa, for pinion of first pair of the cutting part reducer.

Research on the tooth grinding process was carried out when grinding two gears of the same size. Allowance for processing $t = 0.51$ mm. The number of processing stages is 3 (rough $t = 0.24$ mm, semi-finished $t = 0.24$ mm, finished $t = 0.03$ mm).

The results of the wheel grinding process from the zol-gel wheel showed that the tooth grinding power is, on average, $P = 566.5$ W at the semi-clean (2) stage and $P = 400$ W at the clean (3) stage. The accuracy of gear wheel processing corresponds to 2 degrees of accuracy (according to DIN 3962).

The results of research into the grinding process of gear wheels with a circle made of monocorundum showed that the power of tooth grinding is on average $P = 900$ W at semi-clean (2) stages and $P = 800$ W at clean (3) stages. The accuracy of gear wheel processing corresponds to 2 degrees of accuracy (according to DIN 3962).

The results of research into the process of grinding gear wheels with a circle made of chrome corundum (ruby corundum) showed that the power of tooth grinding is, on average, $P = 1100$ W at semi-pure (2) stages and $P = 900$ W at pure (3) stages (Fig. 9). The processing accuracy of the gear wheel also corresponds to 2 degrees of accuracy.



Fig. 9. Installation of a chrome corundum grinding wheel on the machine.

The use of grinding wheels made of monocrystalline corundum (monocorundum and zol-gel corundum) allows you to reduce the effective tooth grinding power 3 times lower than with white corundum wheels and 1.5 times lower than with chrome corundum wheels. Grinding of monocorundum and chrome corundum wheels was carried out through 7–8 depressions of the workpiece, while straightening of a regular wheels of white corundum was carried out through 4–5 depressions.

The study of the surface roughness of gear wheels showed that the roughness when grinding with cubic boron nitride (CBN) wheels provides a R_a value of 1.00–1.10. It depends on the strength of CBN grains, which are much higher than ordinary abrasive materials. When using ordinary abrasive materials, for example, white corundum, R_a 0.50–0.60.

And for example, when using monocrystalline corundum and chrome corundum R_a 0.60–0.70. In all processing modes, an increase in surface roughness is ensured while reducing the processing time of gear wheels. The depth of grinding did not show a significant increase in the surface roughness of the gears.

In the case of complex metallographic and X-ray analysis of the surface layer of gears, it was established that grinding with a cutting depth of $t = 0.1$ mm and more is impractical, because the depth of the defective layer exceeds the allowance for processing and is 200 μm , which cannot be removed during the following finishing processing operations (Fig. 10).

In practice, grinding gears with a depth greater than $t = 0.05$ mm is not advisable, because the defective layer may coalesce after processing. Finishing is appropriate for a grinding depth of up to $t = 0.01$ – 0.015 mm, floor finishing with a depth of $t = 0.03$ mm. With this gear grinding technology, the defective layer is 20–30 microns, which can be removed during the following finishing operations.

Simultaneously with metallographic and X-ray studies, the distribution of final stresses to the depth of the surface layer of gears was studied.

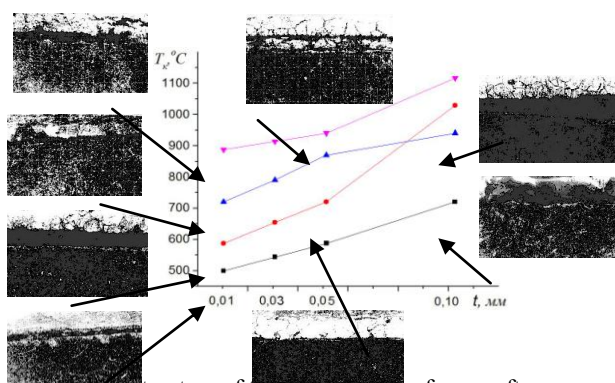


Fig. 10. The structure of the surface layer of gears after grinding with corundum wheels (1, 2, 3, 4) and cubic boron nitride (5, 6, 7, 8).

It was established that at a depth of 5–8 μm , compressive final stresses turn into ultimate tensile stresses, and at a depth of 20 μm they are equal to $100\text{--}400 \pm 10$ MPa. The depth of final tensile stress distribution is 70–100 μm .

The maximum final tensile stresses in the surface layer of gears occur when processing with a tool made of white corundum, and the minimum when processing with a tool made of CBN. During final grinding $t = 0.01$ mm, cubic boron nitride circles are formed by compressing final stresses of up to 200 ± 10 MPa.

It was established that after grinding with CNB wheels, the wear resistance of gear wheels increases by up to 10% compared to grinding with corundum, thereby confirming that due to high-quality processing, it is possible to increase the durability of gear wheels.

Summary

A detailed assessment of the strength and durability of the main parts and components of the cutting part of the new generation coal shearer was carried out. Created and put into serial production of coal shearers at a prominent machine-building enterprise of Ukraine [10].

Practical recommendations for the use of tooth grinding technologies with the purpose and provision of parameters of the surface layer of hardened large-module gear wheels have been developed.

Thus, it can be concluded that the developed design of the gear reducer of the cutting part of the combine provides the necessary durability of 15,000 hours and an average resource before overhaul.

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Flat ultrafiltration PAN/BNE membranes with temperature-responsive performance obtained by modification with “smart” PVA copolymer

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Abstract. The ultrafiltration (UF) composite polyacrylonitrile (PAN)/ butadiene–acrylonitrile elastomer (BNE) membrane obtained by the phase inversion method was modified with temperature sensitivity by physicochemical immobilization of hydrophobically modified poly(vinyl alcohol) (HMPVA). HMPVA was prepared by partial acetalization of poly(vinyl alcohol) (PVA) with a molar mass of 75000 g/mol, resulting in a temperature-sensitive copolymer with a phase transition temperature (T_{tr}) of 40.2°C. Modification of the membranes was performed by introducing aqueous HMPVA solutions of different concentrations into the membrane structure, followed by its immobilization by cross-linking with glutaraldehyde. The composition, technological and structural characteristics of the membranes before and after the modification were investigated. Scanning electron microscopy (SEM) and barodynamic determinations were used in the typical UF operating pressure range (0–0.5 MPa), evaluating permeability (J , l/m².h) and rejection (R , %) of water and albumin as a calibrant (molecular mass ~76 kDa). Temperature-sensitive HMPVA in the membrane structure optimizes the energy and material efficiency of the membrane process, including membrane service life and maintenance.

1 Introduction

Membrane technology is based on separation processes of gas and/or liquid mixtures and constituent components. Membrane technology began its development in the 70s of the 20th century and until now has established itself on an industrial scale in various fields - pharmacy, food and beverage, pulp and paper, etc. The competitiveness of this technology is due to the low capital costs, high technological and environmental efficiency, affordable organization and management of the processes [1]. This makes it adaptable to the principles of the circular economy and sustainable growth and to the trends of Industry 4.0 [2-4].

At the basis of the activity of all membrane processes is the most important element – the semipermeable membrane. The classification of processes and membranes is determined by different signs - structural, morphological, physicochemical, technological, etc. The possibilities for obtaining them are also diverse. This allows different materials to be used alone and as composites, or combining material and membrane synthesis. In addition to the capabilities of modeling the membrane characteristics, a variety of methods and means can be used to modify the forming conditions and/or the materials [5].

Polymers are a conventional material for forming semi-permeable membranes operating under elevated pressure conditions, such as microfiltration (MF),

ultrafiltration (UF), nanofiltration (NF), reverse osmosis (RO). Polyacrylonitrile (PAN) and its copolymers are preferred because of its functionality due to qualities such as relative hydrophilicity, chemical, mechanical and thermal resistance under the operational conditions of membrane filtration and the possibilities of preparation and modification. The quality of PAN membranes obtained by the phase inversion method is determined by the homogeneity, respectively, the composition of the polymer solutions and the conditions under which the coagulation of the polymer takes place during the inversion from liquid to solid state. These are also the main factors on which the structural and morphological realization of the pore distribution in the membranes depends, as a consequence of which are their main technological parameters - permeability and selectivity [6-9].

The membrane structure can be modified in different ways, for example by different additives in the polymer solution or by their incorporation into the finished structure. Modified or not, depending on the technological characteristics and structural specificity of the membranes, they are used in water purification, separation, isolation and fractionation of microorganisms, proteins, etc. [10-12]. Membrane modification can be done with hydrophobically modified poly(vinyl alcohol) (HMPVA), a representative of so-called "smart" materials with temperature-stimulated

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sensitivity, by being introduced and retained in a macroporous membrane structure by cross-linking.

The presented research aims to investigate the possibility of manipulating the performance characteristics of a UF PAN/BNE membrane by introducing and retaining by cross-linking a thermosensitive "smart" HMPVA into the membrane structure. Temperature sensitivity polymers have a phase transition temperature (T_{tr}), according to which the conformational state of the macromolecules changes. This is expressed as the collapse of the polymer chains from elongated helices to collapsed globules and vice versa. The ability for HMPVA to be located in the membrane structure will expand the membrane's performance capabilities related to its permeability and selectivity according to temperature conditions and within the limits critical for stimulating the sensitivity of the "smart" polymer. The choice of the receiving pure (unmodified) membrane and its qualities are of great importance for the result [13-17].

2 Experimental

Polyacrylonitrile fibers obtained from the ternary copolymer poly(acrylonitrile-methylacrylate-2-acryloamide-2-methylpropanesulphonic acid) (PAN) produced in Bulgaria from LUKOIL Neftochim Bourgas Co. and butadiene-acrylonitrile elastomer (SKN-40) (BNE) produced by Omskyi Kautchuk, Russia, were used.

Hydrophobically modified poly(vinyl alcohol) was synthesized according to the method already reported in the literature [18]. Briefly, a solution of PVA in deionized water (2.5 %) was prepared and cooled to 15 °C. Then the corresponding amount of acetaldehyde was added dropwise to the aqueous PVA solution at vigorous stirring while maintaining this temperature with thermostat. The temperature was raised to 30 °C and the reaction was let to proceed at this temperature under vigorous stirring for four hours. The obtained modified PVA was isolated from the solution by precipitation when raising the temperature to 60 °C, purified by repeated dissolution in deionized water and followed precipitation by raising the temperature, and finally lyophilized.

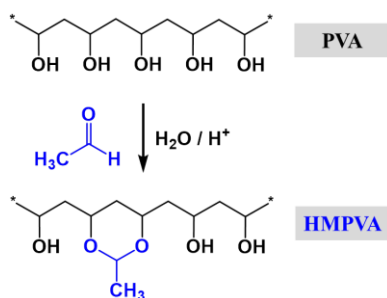


Fig. 1 Synthesis of HMPVA in aqueous solution

Reaction scheme of the hydrophobic modification of PVA is presented in Fig.1. The extend of PVA modification, i.e. the content of introduced acetal groups was determined by NMR spectroscopy and confirmed by Fourier transform infrared spectroscopy (FTIR) analysis

and measurement of the temperature of phase transition - cloud point (T_{CP}) measurement of HMPVA aqueous solution. Characterization of the synthesized HMPVA: substituted -OH groups 17.8 mol % (NMR); acetal content 8.9 mol %; phase transition temperature (cloud point) in aqueous solution 40.2 °C.

NMR analysis was performed on a Bruker Avance DRX 250 spectrometer (Bruker) in DMSO-d₆ as solvent. Phase transition temperature (T_{tr}), also known as cloud points was evaluated by means of UV-Vis spectroscopy by measuring the transmittance of HMPVA aqueous solution at a wavelength of 500 nm as a function of temperature.

The membranes are formed by a dry-wet phase reversal technique from polymer solutions in an aqueous coagulation bath. The polymer solutions were composed of 16.2 wt.% PAN or 16.2 wt.% PAN and 1 wt.% BNE and solvent N,N-dimethylformamide (DMF) product of "Fluka", Germany. The fibers and elastomer are dissolved in DMF when heated to about 70 °C and stirred until the solutions are completely homogenized. After filtration and deaeration, a film was drawn from the polymer solutions onto a calendered polyester mat attached to a glass plate. The polyester mat brand FO-2403, a product of "Velidon Filtren", Germany has a density of 100 ± 5 g/m² and a thickness of 2 ± 0.1 nm. Up to 15 sec. after casting the solution, the plate is immersed in a bath of distilled water at a temperature of 25 ± 1 °C to carry out the phase inversion process and form the membrane structure. The membranes are left for half an hour in the coagulation bath, after which they are washed intensively with water to completely remove the solvent.

The modification of the formed ultrafiltration PAN/BNE membrane was carried out by introducing the thermosensitive polymer into the pore structure of the membrane and immobilized by cross-linking. From each membrane, pieces with an area of 100 cm² were prepared and treated on the side of the selective layer or on the side of the polyester support with a liquid mixture with the following composition: impregnating solution - (1.25g HMPVA in 50ml H₂O), pH corrector (0.1 ml 10% HCl) and cross-linking agent - (0.1 ml 50% glutaraldehyde). For more effective cross-linking, the membranes were first left in a dryer for 30 min at $T=40$ °C and 24 h at room temperature.

Membrane morphology was analyzed from images taken with scanning electron microscope JSM-5510 ("JEOL", Japan). All samples were sputter-coated with gold before analysis. For cross-section analysis, the membrane samples were freeze fractured in liquid nitrogen.

IRAffinity-1 Shimadzu FTIR spectrophotometer equipped with reflectance measuring appliance (MIRacle Attenuated Total Reflectance Attachment) was used to study solid samples of PAN/ HMPVA membranes.

The permeability and rejection of the membranes to water and albumin as calibrant were performed with an ultrafiltration laboratory cell SM 165-26 ("Sartorius", England). The experiments were carried out at different temperatures of 22, 32 and 42 °C which are below and above the T_{tr} of 40.2 °C characteristic for HMPVA. The

permeability for water was studied as function of the pressure ($J=f(P)$). The time for which 10 ml of distilled water passed through the membrane was determined. The time measured at certain pressure was used to calculate membrane permeability by the formula:

$$J = \frac{V}{S \cdot \tau}, \text{ l/m}^2 \cdot \text{h} \quad (1)$$

where: J – flux permeate through the membrane, $\text{l/m}^2 \cdot \text{h}$; V – volume of permeated flux, l; S – effective area of tested samples, m^2 ; τ – record time, h;

The rejection of the membranes were determined at pressure of 0.3 MPa by measuring the time for which 10 ml of permeate of calibrant solution pass through the membrane and calculating by the formula:

$$R = \frac{C_2 - C_1}{C_2} \cdot 100, \% \quad (2)$$

where: R – rejection of the membrane, %; C_2 – concentration of calibrant in the feed, g/l; C_1 – concentration of calibrant in the permeate, g/l;

The concentration of porcine serum albumin with molecular weight 76 kDa („Merck”, Germany) was determined applying the calibration curve method. UNICA 8625 UV-Vis spectrometer (France) was used and absorption of albumin solutions at the wavelength $\lambda=280$ nm was measured against 0.9 wt.% NaCl solution (blank test)

3 Results and discussion

The leading conditions in choosing a suitable membrane for modification by physicochemical immobilization of the thermosensitive polymer is that it has a structure with volume pores and mechanical stability. Such a membrane was selected from PAN/BNE membranes developed in other studies [19,20]. It has been proven that, depending on the amount used as an additive in PAN polymer solutions, BNE structurally modifies the membranes in the desired direction. The selected membrane with the relevant qualities for the purpose of the study was formed from a solution with a composition of 16.2 wt.% PAN, 1.0 wt.% BNE in DMF solvent. To visualize the morphological differences of the asymmetric structures of PAN and PAN/BNE membranes, scanned images are presented in Fig. 2. From the cross-sectional image of the PAN/BNE membrane, it is evident that the pores in the drainage sublayer are more voluminous compared to the PAN membrane with the same polymer concentration. This layer primarily provides the amount of flux passed through the membrane per unit time.

The morphological differences in the structure of the membranes (Fig. 2) are reflected in the water permeability values, which are represented by the hysteresis curves in Fig. 3. They are constructed as $J=f(P)$ when increasing and decreasing the pressure by 0.1 MPa in the range of 0 to 0.5 MPa and applied perpendicular to the water flow and the membrane. The change in pressure ensures the inclusion and exclusion of pores of different sizes in the membrane filtration process. From the course of the

curves, the change in permeability depending on the pressure is followed, and from the size of the hysteresis area limited between them, the mechanical stability of the membrane structure under the operating conditions can be judged.

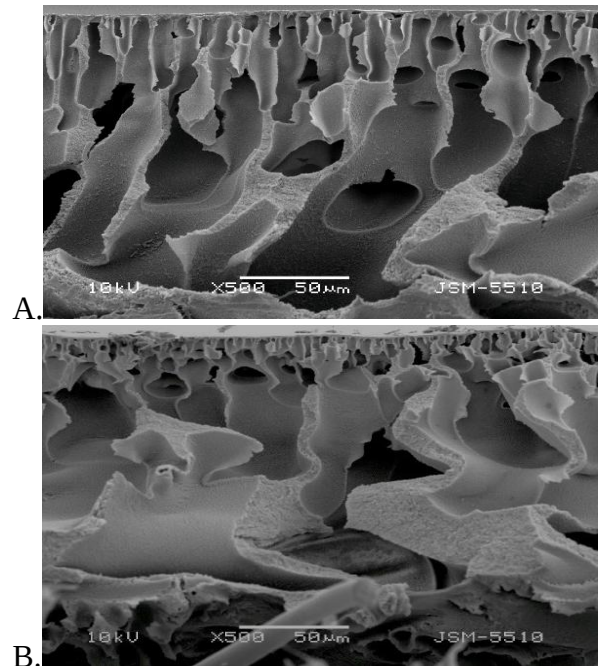


Fig. 2. SEM images of a cross-section of PAN (A) and PAN/BNE (B) membranes.

According to the presented results (Fig. 3), both membranes have an optimal ratio of hysteresis area, but the one chosen as the object of the target modification has a much higher water permeability up to 4 850 $\text{l/m}^2 \cdot \text{h}$ at maximum pressure. Undoubtedly, these values are determined by the morphological differences (Fig. 2) caused by the different polymer composition, as all other conditions related to the preparation are identical.

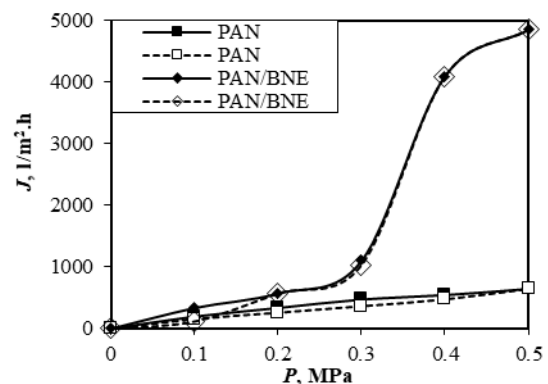


Fig. 3. Hysteresis curves of water flux permeability of PAN and PAN/BNE membranes.

The technological characteristics of the membrane were measured at different temperatures – 22, 32 and 42 °C, in order to pre-test its behavior according to the temperature conditions stimulating the sensitivity of the modifying HMPVA. The results for the receiving (base-

line) membrane specified for the modification are reflected in Fig. 4 and Table 1.

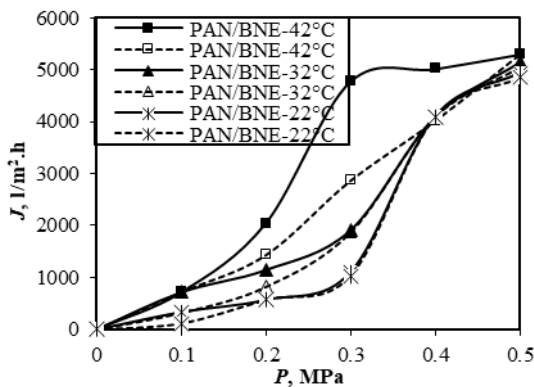


Fig. 4. Hysteresis curves of water flux permeability of PAN/BNE membranes at different operating temperatures.

The structure is affected by temperature and pressure. The membrane exhibits water permeability and hysteresis area that increase proportionally with increasing temperature. According to this, at $T=42^\circ\text{C}$ the structure has the lower mechanical stability compared to the lower temperatures. Selectivity and permeability to albumin under the same conditions follow the same dependence on the influence of temperature. It should be noted that, in principle for a UF process, selectivity and permeability are inversely related [21-23].

Table 1. Technological characteristics of the membranes determined toward albumin at different temperatures (working pressure 0.3 MPa).

Temperature (°C)	Membrane characteristics	
	J_{alb} (l/m ² .h)	R_{alb} (%)
22	101	25
32	143	20
42	184	18

After impregnation of the membranes on the side of the selective layer and on the side of the polyester macroporous support with a temperature-sensitive polymer, it was found that a positive result was obtained only in the second variant of treatment. This reinforces the assumption that the macroporous structure enables easier access of polymer into it.

The modification of the membranes with HMPVA polymer is carried out with different volumes of impregnating liquid mixture with concentrations of HMPVA, respectively: 10 ml - 0.25 wt.%, 2.5 ml - 0.06 wt.%, 1 ml - 0.03 wt.%. When cross-linking with the solution with the highest concentration, the membrane structure is compacted and the water permeability is significantly reduced (Fig. 5).

A cross-linked HMPVA polymer film was also observed on the side of the selective surface opposite to the treated substrate. This is indicative of an excess of impregnating mixture and the cause of drastically reduced water permeability compared to the unmodified membrane (Fig.4 and 5).

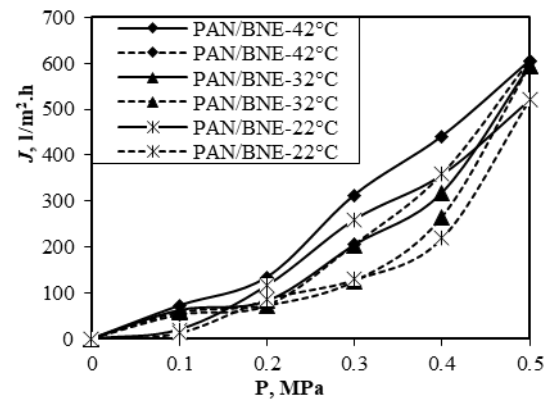


Fig. 5. Hysteresis curves of water flux permeability of a membrane modified with 0.25 wt.% HMPVA at different operating temperatures.

Subsequently, smaller amounts of the cross-linking mixture were used, which made the penetration and distribution of HMPVA in the structure of the membranes more efficient. From the hysteresis curves for the respective membranes in Figs. 6 and 7, an increase in the water permeability values is observed, and at 0.06 wt.%, an optimal compaction of the pore structure was also achieved. The same membrane has smaller hysteresis areas at all three temperature regimes (Fig.6), compared to the unmodified (Fig.4) and modified with 0.25 wt.% (Fig.5) and 0.03 wt.% HMPVA. The modifying temperature-sensitive effect of HMPVA regarding the permeability and selectivity of the membrane structure is also very well expressed. The water permeability of the membrane remarkably changes according to the conformational change of HMPVA in accordance with the change of temperature relative to the T_{tr} and applied pressure. At the highest pressure, the water permeability of 573 l/m².h at a temperature below the T_{tr} increases to 1147 l/m².h at a temperature above the T_{tr} . The changes are the result of the reaction of HMPVA as a result of its physicochemical binding to the membrane structure and evidence of its effective retention according to the methodology carried out.

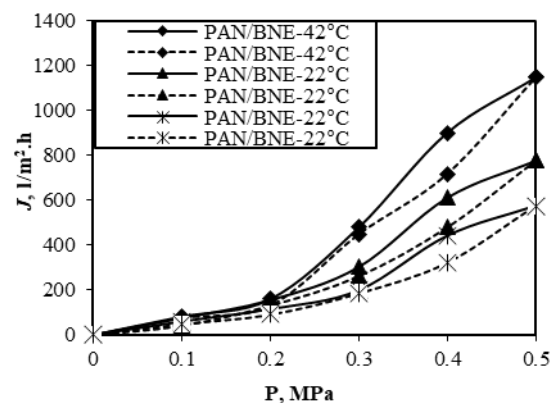


Fig. 6. Hysteresis curves of water flux permeability of a membrane modified with 0.06 mass% HMPVA at different operating temperatures.

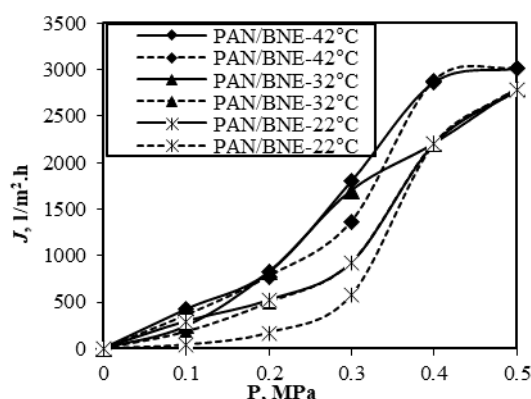


Fig. 7. Hysteresis curves of water flux permeability of a membrane modified with 0.03 mass% HMPVA at different operating temperatures.

Confirmation of the modifying importance of HMPVA, expressed in sharing its temperature sensitivity with the structure of the PAN/BNE membrane, are the permeability and selectivity values of the membranes reflected in table 2. Regarding the selectivity of the membranes, the results also change in proportion to the temperature change when using 2.5 ml and 1.0 ml of HMPVA impregnating solution. The difference in selectivity values below and above T_{tr} is more significant with the smallest immobilized amount of HMPVA. This explains the possibility of the polymer penetrating the finer membrane structure of the selective layer. The configuration of HMPVA in the membrane matrix shows stability in the temperature range where the differences in the permeabilities and selectivities of the membranes are probably due to the change in the behavior of the polymer membrane matrix. At temperatures above 40.2°C (T_{tr} of the polymer), the collapse of the HMPVA polymer creates conditions for high permeability but low selectivity.

Table 2. Technological characteristics of the modified membranes determined toward albumin at different temperatures (working pressure 0.3MPa).

HMPVA content	Membrane characteristics			
	at 22°C		at 42°C	
	J_{alb} ($\text{l/m}^2\cdot\text{h}$)	R_{alb} (%)	J_{alb} ($\text{l/m}^2\cdot\text{h}$)	R_{alb} (%)
10 ml 0.25 wt.%	25	65	76	24
2.5 ml 0.06 wt.%	34	57	56	23
1ml 0.03 wt.%	57	49	717	3

The modification with 2.5 ml of impregnating solution with a HMPVA concentration of 0.06 wt.% is optimal. The imputed temperature intelligence in the PAN/BNE membrane, by immobilizing HMPVA in its structure, makes it technologically attractive for UF application. It could be successfully applied to separate

liquid mixtures under temperature conditions with and without the stimulation of temperature "intelligence". Depending on these conditions, the membrane will exhibit the corresponding performance characteristics. This will optimize the energy and material efficiency of the membrane process regardless of the purpose - purification, separation, concentration or others, including the operational life and maintenance of the membrane.

4 Conclusions

It was found that UF composite PAN/BNE membrane can be modified with temperature sensitivity by physico-chemical immobilization of HMPVA introduced as an impregnating solution on the side of the membrane polyester support. An optimal temperature-stimulated reaction, according to the T_{tr} of HMPVA (40.2°C), definitely exhibits the membrane modified with 0.06 wt.% HMPVA. It has been proven to change according to the T_{tr} its technological characteristics - for the permeability to water from $573 \text{ l/m}^2\cdot\text{h}$ to $1147 \text{ l/m}^2\cdot\text{h}$ and to the calibrant from $34 \text{ l/m}^2\cdot\text{h}$ to $56 \text{ l/m}^2\cdot\text{h}$ at the selectivity of 57% of 23%. The most effective structure-modifying amount of HMPVA also improves and the mechanical resistance of the membrane.

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Load carrying capacities of gears with a lattice structure body

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Abstract. Lattice structures are type of topology structures that have complex geometry, composed of multiplied unit cells through which a pattern is generated. Lattice structures are of great interest in engineering due to their strength-to-weight ratio. There has been an increasing trend for their application as infill patterns in a variety of engineering parts and elements. However, the complexity of the lattice geometries, makes them difficult to be produced by conventional methods. Therefore, additive manufacturing technologies have been used as technologies for production of parts containing lattice structures. In this research, the focus is placed on analyzing various unit cell structures and their application in conventional gears as their structure body. One specific lattice structure is chosen and generated. Several characteristics of the lattice structure can vary, like the cell size, density, wall thickness etc. The lattice shape will remain the same for all the analysis. The lattice is optimized by weight reduction and maintaining load carrying capacity of the gears. Different samples are examined using FEM (Finite Element Method) in terms of determination the load carrying capacity. The results for the optimized gear body structures are elaborated, conclusions are drawn and recommendations for application of gears with a specific lattice structure are provided.

1 Introduction

Cellular or lattice structures and additive manufacturing (AM) represent a powerful combination that has transformed the way we design and manufacture objects. These technologies offer immense potential for creating lightweight, strong, and functionally optimized components across a wide range of industries.

Cellular structures are abundant in nature and can be found in various organisms and materials, such as the honeycomb structures, sponge skeletons, bone trabeculae, plant parenchyma cells, etc. By drawing inspiration from these natural cellular structures, engineers have been developing structures and materials that exhibit similar characteristics. Foams are one type of modern technical materials that have a cellular architecture. Metal foams, for example, are generated using alloy combinations, treating them with regulated heating processes and including a pre-treated foaming agent that releases gas [1]. As explained by [1], these type of metal foams are ideal for lightweight constructions due to a high stiffness-to-mass ratio compared to the corresponding dense materials. However, such cellular materials result with cell characteristics that are randomly obtained through the described processes, with no control over the cell sizes and arrangements [2]. Lattice structures, on the other hand, are a type of cellular structures which differ from foams due to their precise arrangement and periodic geometry that has a regularly repeating structure of the

unit cell that it is composed of [2, 3]. Therefore, lattice structures are “architected cellular materials” that allow high control over their cellular structure and as a result, provide a method to generate the desirable mechanical properties of the material by simply modifying the geometry of the lattice (or lattice units) [2].

Lattice structures provide unique properties due to their geometry. The lattice framework allows for efficient material usage and is very light-weighting. These structures exhibit excellent mechanical and thermal properties, such as high strength-to-weight ratio, high strength and stiffness, improved energy absorption, enhanced load distribution, and high permeability [2, 3]. These properties make them ideal for application in multiple industries: biomedical, aerospace, sports, etc. The mechanical properties of the lattice structures are linked to (1) the mechanical properties of the parent material and (2) the type of lattice. The relative density of the lattice, which is expressed as a ratio of the density of the lattice and the density of the parent material, is directly dependent with the mechanical properties – as the relative density decreases, the mechanical properties of the lattice structure also decrease [3]. The mechanical properties also depend on the structure of the unit cells – “the size, type, orientation, and boundary conditions of the periodic unit cell” which can be designed to obtain specific mechanical properties [4].

Regarding the different types of lattices, that consist of different types of unit cells, they are mainly classified in three families: surface-based – generated

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from trigonometric equations, strut-based – consisted of rods (struts, or beams) and nodes and planar based – created from extruded 2D planes [5]. Strut-based lattices have a simpler design and are defined by the Maxwell number (M) that depends on their number of struts and rods and if the M number is positive the structure is stretch-dominated (meaning that it is stiff and strong) [2, 3]. The surface-based lattices are triply periodic minimal surfaces (TPMS) and are bending-dominated, designed to distribute loads and forces across the lattice structure, effectively transferring bending stresses throughout the lattice framework. The most commonly used TPMS lattice is the gyroid which has a spherical core and circular, smooth struts, and is self-supporting [6, 7]. TPMS lattices, such as the gyroid, have potential advantages over other types of lattice structures. The presence of continuous curvature in gyroid lattices and absence of nodes in these architectures decreases stress concentration, leading to improved fatigue strength [2]. Gyroids exhibit nearly triple the specific energy absorption compared to strut-based structures of similar porosity [3]. In addition, TPMS lattices are easier to manufacture as a result of their continuous structure, which is another key benefit, especially since lattice structures are challenging to produce.

The complexity of the lattice geometries makes them impractical and impossible to be produced by conventional methods and they can be accurately produced only by additive manufacturing (AM) technologies. Unlike traditional subtractive manufacturing techniques that involve removing material from a larger block, AM builds physical objects from digital information layer-by-layer, a process which provides many design opportunities and benefits like: creation of complex, internal, freeform geometries; production of macro-structure topology optimized objects for reduced material; and creation of three-dimensional lattices and trusses with specific properties [4]. By using AM technologies, such as Powder Bed Fusion (PBF), Selective Laser Melting (SLM) or Electron Beam Melting (EBM), metal lattice structures for mechanical components can be efficiently fabricated. The processes vary, but they are all based on melting layers of metal powders. However, the precision of production also depends on the type of lattice. Research shows that lattice structures with gyroid unit cells have best manufacturability. As described by [7], “the inclination angle of the circular and smooth struts of the gyroid unit cell continuously varies along the spherical core, which makes layers grow up gradually” and “the previously manufactured layer can support next layer, the gyroid lattice structures are self-supporting”. In the research, using SLM process, gyroid lattice structures with a volume fraction of 15% and unit cell sizes ranging from 2 to 8 mm were successfully manufactured without any noticeable deformations. In another research [8], two gyroid lattices (Schoen Gyroid and Schwartz Diamond) were produced using SLM to investigate the manufacturability of the gyroid units with different cell sizes (2, 5 and 8mm). Results showed that all sizes were manufacturable through SLM – the 8 mm cell size with no deformation and the 2 mm cell size with no damage

of struts. The fact that small cell sizes can be produced with no damage is a crucial aspect since smaller lattice cells have higher densities and relative densities and higher yield strength and Young’s modulus, thus better performance [7].

This newfound ability to generate complex design of mechanical components and produce them successfully with suitable AM methods has been highly advantageous for enhancing product performance across a wide range of applications.

In this research, our aim is to contribute to the expansion of knowledge regarding the methods of incorporating these lattice structures into machine components intended for larger machine assemblies, with the main goal of reducing the weight of the parts while maintaining the required mechanical characteristics. In this study, it was decided to apply surface-based, TPMS, gyroid lattice due to all the aforementioned advantages and to experiment with the characteristics of its unit cells, analyze the differences, and provide recommendations for implementation. The main research objectives are elaborated in greater detail in the following section.

2 Objectives

This research focuses on the application of lattice structures in conventional gears as their structure body. The emphasis is placed on optimizing the lattice structure for weight reduction while maintaining the load carrying capacity.

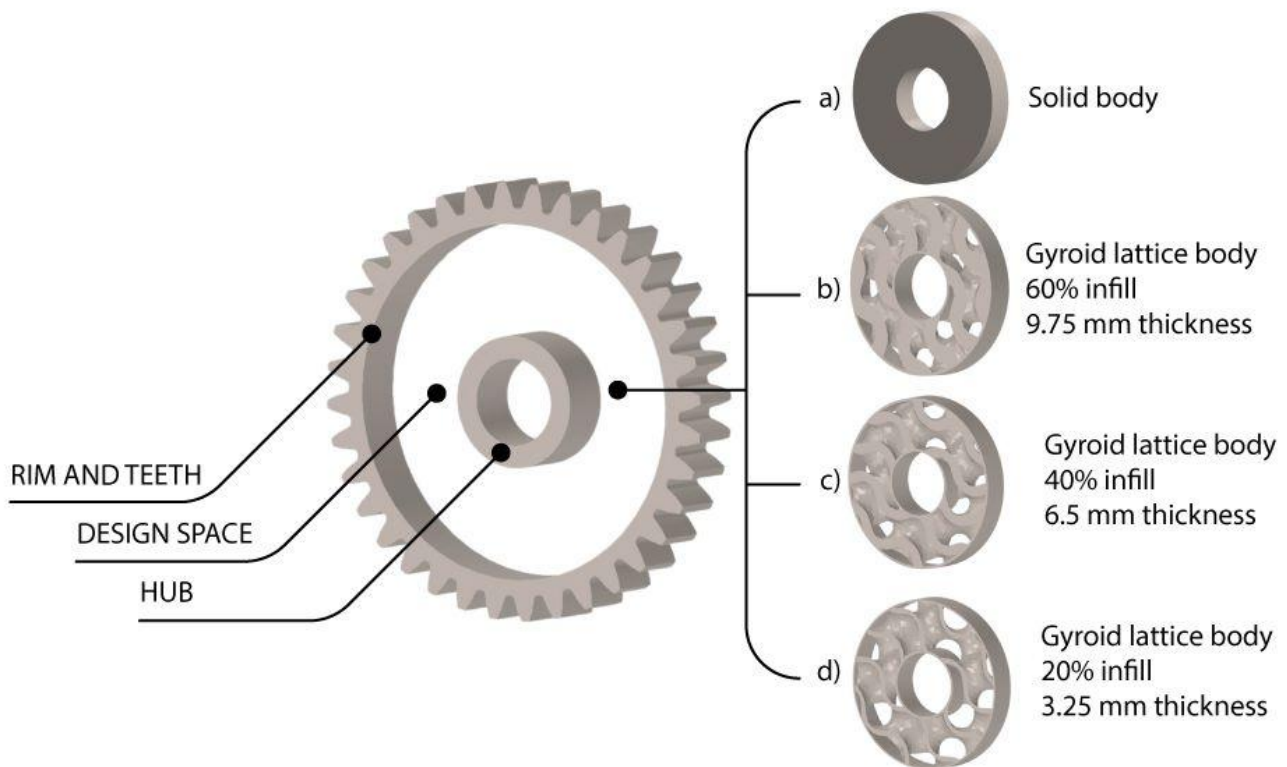
Reducing the weight and vibration of gears has been an important goal which leads to achieving improved performance, efficiency and longevity of gear systems. There are numerous researches that experiment with lattice structures to achieve this goal. Mainly, the application of lattices in gears is done through processes of topology optimization, according to Finite Element Analysis (FEA), which means optimizing the application of the lattice based on previously analyzed stress and strain distribution [9]. Another method is firstly selecting the lattice topology and replacing the solid with the lattice structure based on previous knowledge, continuing with homogenization of the lattice cell with the use of FEA, then applying a genetic algorithm to help define the strut diameter value for each cell, and in the end, evaluating obtained solutions, again using FEA [10]. For example, in the research of [11], a Ti spur gear with a cellular structure between the gear toothed ring and fixed hub, generated using a topology optimization software, was manufactured by the SLM process. The results showed reduced gear vibrations with the new gear design. The research of [12] presents a similar experimental approach for optimizing gears where a topology optimization software was applied to generate a lattice gear body, but the lattice was optimized further to remove stress concentrations and to reduce the stress levels. The research of [13], on the other hand, is based on the Michell Truss Method, rather than only a topology optimization, to design lightweight gears.

Although such researches in the field of lightweight gears cover the analysis of topology optimization methods and present performance results of the generated lightweight lattice gears, there is very limited information regarding the application of different types of lattice unit cells in gears, as well as cell variations. Therefore, this study aims to explore one specific lattice structure chosen for generating a lightweight gear body and the effects of changing the variable characteristics of that lattice. For this research, the gyroid lattice is chosen due to the multiple advantages it offers, as elaborated in existing literature, all of which are of great importance for the construction of gears: outperforming strut-based lattices on angular loads; better distribution of load; lack of stress concentration regions; increased fatigue life; etc. [14]. The characteristics of the TPMS, gyroid lattices were stated in various researches, as given in the introduction part, but the application of such a lattice precisely in gears was found in very little sources. The most similar approach to the one used in this research was found in the paper of L. H. Nguyen and K. T. Nguyen, 2022 [15] – where the mechanical properties of the gyroid structure applied to additive-manufactured plastic gears are explored. This research focuses on properties of metal gears, that can be produced by SLM.

The main objective of this study is to vary the approximate thickness, or unit cell thickness, of the Gyroid Faced Lattice applied as an infill in the gear body, and compare samples using FEM analysis to conclude which cell configuration results with maximum

3 Methodology

To assess the impact of lattice structures on the load carrying capacity of gears, a comprehensive analysis of four cases was conducted. Each case involved a different gear configuration, ranging from a solid gear, to gears with lattice structures of varying infill percentages and thicknesses. The first case served as the baseline, using a solid gear design. This traditional approach provided a reference point for comparing the subsequent cases. It represented a gear with a continuous, solid structure, without any lattice patterns or voids. Furthermore, the second case featured gears with a lattice structure, consisting of 60% infill and a thickness of 9.75 mm. In this configuration, the gear incorporated a lattice pattern with a relatively high infill percentage, ensuring a substantial amount of material within the structure. The increased thickness of 9.75 mm aimed to maintain the overall strength and rigidity of the gear. The third case, explored gears with a lattice structure comprising 40% infill and a thickness of 6.5 mm. This design sought a balance between weight reduction and structural integrity. By reducing the infill percentage and thickness, compared to the second case, the gear aimed to decrease its weight. In the fourth and final case, gears with a lattice structure featuring 20% infill and a thickness of 3.25 mm were examined. This design emphasized maximum weight reduction while still ensuring the gear's functional performance. The lower infill percentage and thinner lattice structure contributed



gear weight reduction while not compromising the required load carrying capacities.

The used methodology, procedure, results and conclusions are elaborated in the following sections.

to a significant reduction in weight compared to the previous cases. By analyzing these four cases, the aim was to understand how the implementation of lattice

Fig. 1. Design space allocated for implementing the gyroid lattice structure in the gear designs and variations between the four different cases.

structures with varying infill percentages and thicknesses impacts the load carrying capacity of gears.

Figure 1 illustrates the design space for implementing the lattice structure in the gears and shows the variations between the different infill amount and wall thickness.

All four cases of the analysis include gears with identical characteristics. The gears that are used, share same specifications, like module of 4 mm, pressure angle of 20 degrees and width of 20 mm. These consistent parameters ensure uniformity in size, tooth profile and gear geometry among all cases. The standardized characteristics neglect the gear production factors and include only the effects of the lattice structures on the load carrying capacity of the gears. Detailed specification of the gears parameters is given in Table 1.

Table 1. Specification of gear set.

Parameter	Pinion	Gear
Number of teeth	36	36
Modul [mm]	4	4
Pressure angle [°]	20	20
Pitch Diameter [mm]	144	144
Face Width [mm]	20	20
Material Young's module [N/mm ²]	206 000	
Poisson's ratio	0.3	
Torque [Nm]	150	
Contact force [N]	2083	

FE analysis was conducted to assess the effects of the changed lattice structure on the load-carrying capacity of the gears. The focus were parameters such as contact pressure, root stress, stress in the body of the gear and total deformation.

Furthermore, an analytical calculation of the contact pressure was conducted following the guidelines outlined in ISO 6336-2:2006. This calculation was carried out to verify and validate the simulation results obtained from the FEA. According to ISO 6336-2:2006, the formula for calculating the nominal contact stress at the pitch point P, for flawless gearing (ideal gears, without errors), and because of the application of static nominal torque, is as follows [16,17]:

$$\sigma_{H0} = Z_H Z_E Z_\epsilon Z_\beta \sqrt{\frac{F_t}{b d_1} \frac{u+1}{u}} \quad (1)$$

Where,

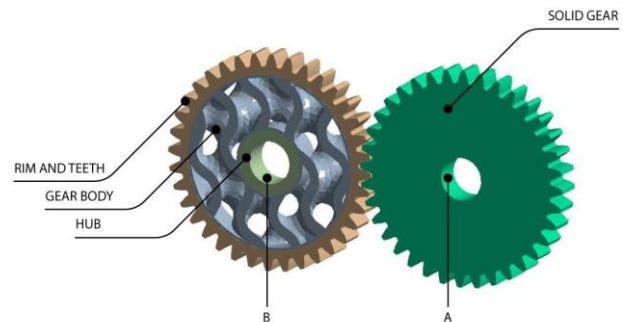
- Z_H - Zone factor
- Z_E - Elasticity factor
- Z_ϵ - Contact ratio factor
- Z_β - Helix angle factor
- F_t - Nominal tangential load
- b - Face width
- d_1 - Pitch diameter
- u - Gear ratio

Detailed information on the calculation procedure for contact pressure can be found in [17].

3 FEM Analysis

FEM was conducted to understand the mechanical behaviour of involute spur gears with lattice structure bodies and varying lattice infill percentages. Four cases were analysed, which included: a solid gear, a gear with a lattice structure and 60% infill, a gear with a lattice structure and 40% infill, and a gear with a lattice structure and 20% infill. Figure 2 illustrates the gears in contact, where one gear was used as the test gear and its body was changed for every case, while the other gear remained solid and was used as contact gear. The 3D models for all cases were created using SolidWorks and Ansys Spaceclaim. After preparing the 3D models, they were imported into Ansys Workbench for the FEM analysis.

For all cases, consistent contact and boundary conditions were used. Bounded contact was defined between the Gear body and Rim, and between the Gear body and Hub, as shown in Figure 2. Additionally, frictionless contact was specified between the gears. Boundary conditions were applied to both gears in the analysis. A cylindrical support with no degree of freedom was defined on surface A (Figure 2), representing a fixed connection. On surface B (Figure 2), a cylindrical support with free rotation was defined, allowing the gear to rotate freely around its central axis.



Additionally, a moment of 150 Nm was applied to surface B (Figure 2) to represent the applied load on the gears.

Fig. 2. Gears in contact (case with 40% infill).

A well-structured mesh is crucial to obtain accurate stress and deformation results in FEM analysis. Initially, an automatic mesh was created for the gears in all four cases. However, due to the complex geometry of the lattice structure, Ansys mesh control options were included to define a finer mesh in the contact zones between the gears and areas where stress concentrations were expected. This refined mesh was used to ensure precise stress analysis, particularly in critical regions. Because of the complexity of the lattice structure,

tetrahedral elements were chosen for mesh generation, as shown in Figure 3. Tetrahedral elements are well-suited for handling complex geometries in FEM analysis.

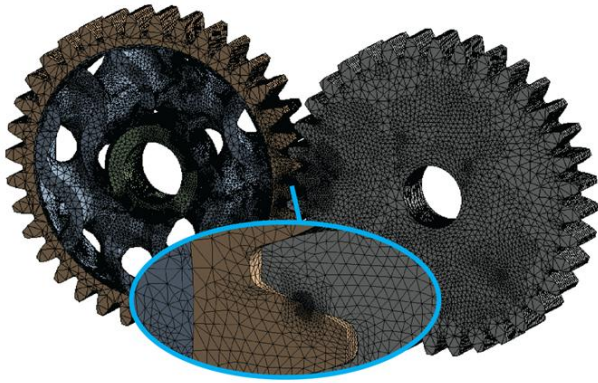


Fig. 3. Mesh with tetrahedral elements for gears in contact (case with 40% infill).

4 Results and discussion

To investigate the influence of the lattice structure, we performed and analyzed several results, including root stress, body stress, contact pressure, and total deformations. An analytical calculation for the contact pressure was conducted and 500.2 MPa was obtained, which was used to verify the results of the FEM analysis. The contact pressure is calculated based on equation (1):

$$\sigma_{H0} = Z_H Z_E Z_\epsilon Z_\beta \sqrt{\frac{F_t}{b d_1} \frac{u+1}{u}}$$

$$= 2.49 \cdot 189.8 \cdot 0.88 \cdot 1 \cdot \sqrt{\frac{2083}{20 \cdot 144} \frac{1+1}{1}} = 500.2 \text{ MPa}$$

The FEM analysis for the first case, involving a solid gear, provided a contact pressure of 498.69 MPa, resulting in error of -0.3% compared to the analytical calculation.

All the results from the FEM analysis are presented in Table 2, while Figures 4 to 8 show the outcomes for Case C, which involves a gear with a lattice structure body and 40% infill. As expected, there were no significant changes in contact pressure and root stress. However, substantial changes were observed in the stress within the gear body. For instance, the stress increased from 8,595 MPa for the solid gear, to 42,635 MPa for the gear with a lattice structure body and 60% infill, 136.45 MPa for 40% infill, and 514.24 MPa for 20% infill. Total deformation also showed variations, ranging from 0.013 mm for Case A, 0.0159 mm for Case B, 0.0175 mm for case C to 0.0245 mm for Case D.

Furthermore, the mass of the gears changed significantly, which is a big advantage for using lattice structures. While a solid gear weighs 2441.3 g, Case D exhibited a decrease to 1261.9 g, representing a 48.31%

reduction. Figure 8 illustrates the comparison between mass reduction and stress increase in the gear body. As a result of the significant mass reduction, the body stress increased considerably. The results imply that in Case D, damage would likely occur in the gear body rather than in the root of the tooth or on the contact surface of the gears.

Table 2. FEM Analysis results.

	Root Stress [MPa]	Body Stress [MPa]	Contact Pressure [MPa]	Total Deformation [mm]	Gear Mass [g]
Case A	63.922	8.595	498.69	0.013	2441.3
Case B	70.454	42.635	486.25	0.0159	1839.5
Case C	70.188	136.45	480.87	0.0175	1570.33
Case D	71.689	514.24	481.71	0.0245	1261.9

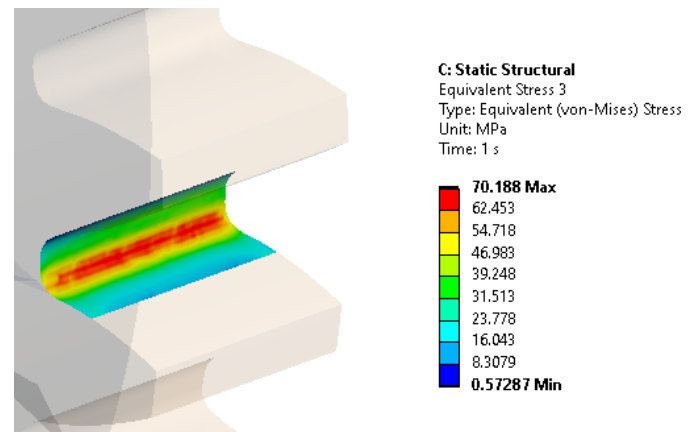


Fig. 4. Root stress (Case C)

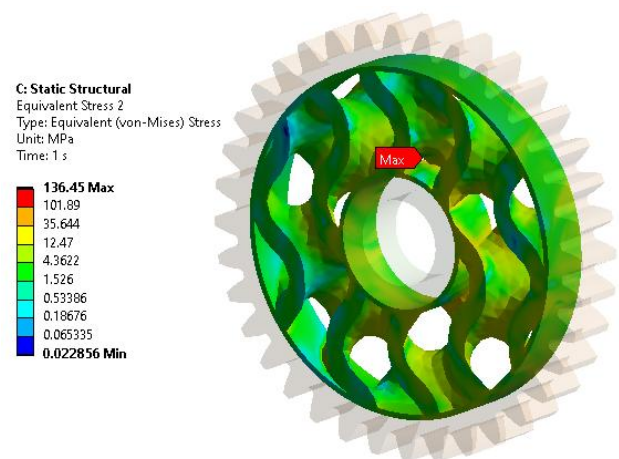


Fig. 5. Body stress (Case C)

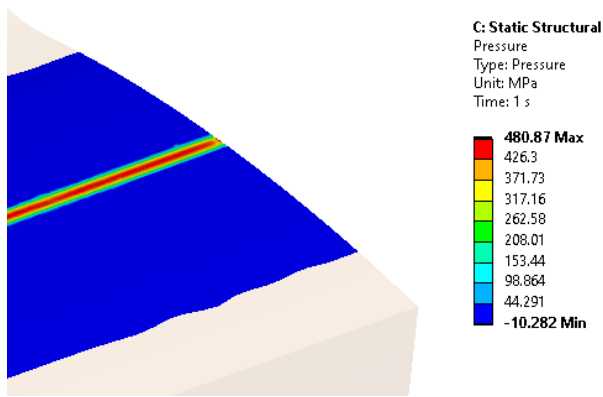


Fig. 6. Contact pressure (Case C)

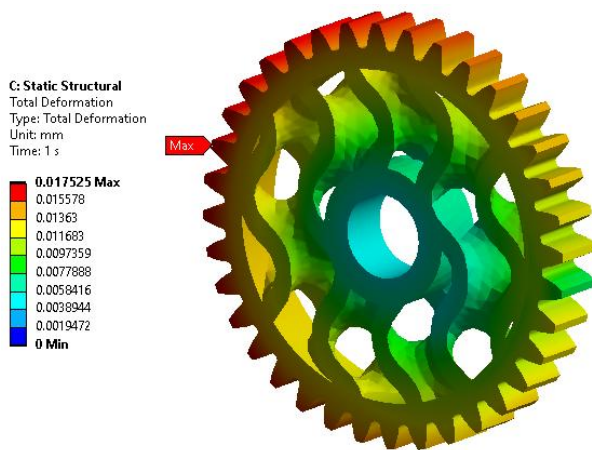


Fig. 7. Total deformation (Case C)

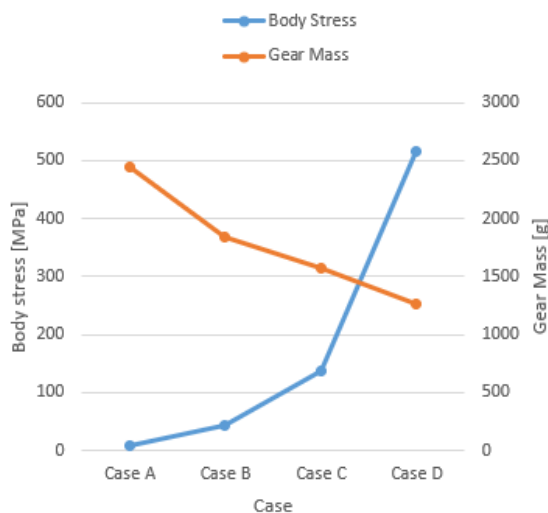


Fig. 8. Comparison between gear mass and body stress

5 Conclusion

Usage of lattice structures in gears offers significant advantages in terms of mass and vibration reduction. However, it is crucial to ensure that the load-carrying capacity of the gears is not compromised. In

this study, various cases were analyzed, and the results indicate that reducing the infill percentage in the lattice structure significantly reduces the gear's mass. This reduction comes at the cost of a substantial increase in stress within the gear body, both root stress and contact pressure. Consequently, the gear body becomes more exposed to failure compared to the root of the teeth or contact surface.

As expected, the root stress remains relatively unaffected, as there were no changes in the rim's thickness during the analysis. Future research should investigate the influence of rim thickness on root stress, determining the minimum rim thickness required for gears with a lattice structure body. Such analysis would provide valuable insights for optimizing the design and performance of gears with lattice structures.

The use of lattice structures offers weight and vibration reduction. However, designers must carefully consider the trade-off between mass reduction and increased stress in the gear body. By addressing these concerns and conducting further research, gears with lattice structures can be designed to achieve optimal performance and durability.

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RESEARCH OF THE POSSIBILITY OF USING FILTER MATERIALS WITH ORTHOTROPIC STRUCTURE BASED ON WOVEN NETS FOR CLEANING DIESEL FUEL

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Abstract. A comparative analysis of the characteristics of experimental samples of filter materials with an orthotropic structure based on woven meshes (FMTS) and standard paper filters was carried out, which confirmed the possibility of using filter elements from FMTS for fine purification of diesel fuel in automotive equipment. Promising directions for further research in the development of effective FMTS are given.

1 Introduction

The machinery and equipment of modern enterprises of the agro-industrial complex cannot do without cleaning liquid and gaseous media with filter materials (used as participants in the technological cycle in the manufacture of products or consumer goods), which ensures the achievement of the required quality of the products produced with their help or the trouble-free operation of machines and mechanisms [1, 2]. It was shown in [3] that a promising material for filtering liquids and gases is a filter material with an orthotropic structure based on woven meshes: while maintaining the advantages of traditional mesh materials (combination of high strength and permeability, stability of the porous structure, which excludes the migration of FM particles into the medium high heat resistance, ability to multiple regeneration), FMTS has 100% regenerability (when the filter element is disassembled, the circuit of the elementary filter cell becomes open, and contaminants on the filter material of the filter element are retained only by adhesion forces), and a spontaneous change in the cell size in the light during regeneration does not affect the fineness cleaning. In case of urgent need in case of repair or maintenance, the product from FMTS, in the presence of original meshes, can be manufactured at any mechanical site using available tools. In addition, in case of loss for any consumer properties reason, the filter material with an orthotropic structure based on woven meshes can be returned to production without any problems by using it as a raw material at metallurgical enterprises.

At the moment, a fairly large amount of work has been done in the FMTS development: promising designs of filter materials with an orthotropic structure based on woven meshes have been described, a material model has been developed with a simple regular structure (or FMTS with a simple laying of meshes, when mesh layers are stacked on top of each other in the same spatial

orientation), on the basis of which equations were developed for calculating the structural and hydrodynamic properties [3]; the possibilities of using 3-D modeling and 3-D printing to study the properties of FMTS, in particular, with the structure described above, are disclosed, which can significantly reduce the complexity of the research process [4, 5]; on the basis of FMTS, a filter for coarse cleaning of herbicides and a drainage element for the catalytic treatment unit in a water deironing plant were developed, manufactured and implemented [3, 6]; a systematic approach has been developed for compiling an experiment planning matrix to establish the relationship between technological characteristics and FMTS properties[7].

The work aim is to substantiate the possibility of using filter materials with an orthotropic structure based on woven meshes for diesel fuel purification.

2 Experimental results and discussion

To substantiate the possibility of using filter materials with an orthotropic structure based on woven meshes for cleaning diesel fuel, a regular (paper) filter DIFA 6101/1 used in the fuel supply system of D-240/245 autotractor engines was chosen as an analog for comparing characteristics. Figure 1 shows the DIFA 6101/1 filter assembled and disassembled.

Then, by the method of displacement of liquid (ethyl alcohol) according to GOST 26849 86, the dimensions of the maximum pores of the filter element installed in the standard filter were studied and, based on the results of the studies, the FMTS was selected for the manufacture of an experimental sample on its basis with the same pore sizes (experimental samples of the FMTS (EO) are a package of grids in the form of a rectangular parallelepiped with a height of 50 mm, having a square with a side of 40 mm at the base.) To study the properties of EO, a tooling was made,

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a 3-D model of which, assembled with a sample, is shown in Figure 2.



Fig. 1. Filter DIFA 6101/1 assembled and disassembled

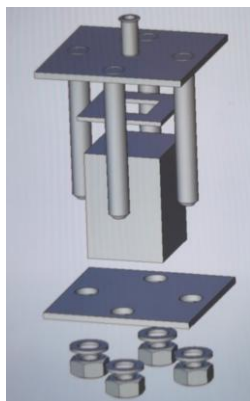


Fig. 2. 3-D sample fixture model

Figure 3 shows the tooling in disassembled form (in the process manufacturing, it was decided to change the location of the holes) and the original woven meshes, in Figure 4 - tooling assembly with EO FMTS, and in Figure 5 - the process of determining the maximum pore sizes on a standard filter element and EO FMTS.



Fig. 3. Exploded rigging and original woven meshes

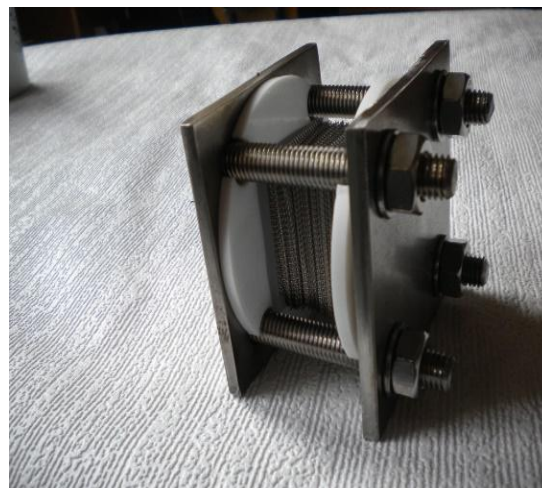


Fig. 4. Tooling assembly with EO FMTS



a)



b)

Fig. 5. Determination of the maximum pore sizes on a standard filter element (a) and EO FMTS (b)

Then, according to GOST 25283-93, the viscosity coefficient of permeability of EO FMTS was determined, which made it possible to calculate its

throughput. As a result, it was found that the characteristics of the experimental sample from FMTS correspond to the conditions of use for its intended purpose: with the same cleaning quality (estimated by pore size) with a standard filter element, FMTS provides the required fuel consumption (7.5 l/h) when converted to the filtration area, corresponding to the volume of the standard filter housing. That is, as a result of the research, the possibility of using FMTS filter elements for cleaning diesel fuel in agricultural machinery was confirmed. The next stage of work is planned full-scale tests of a prototype in the field.

At the same time, in the process of conducting research, a range of tasks was determined to be solved in the field of developing FMTS and products made from them. So, in the manufacture of an experimental sample, it was found that, although the manufacture of a sample from FMTS with the simple regular structure described above (Figure 6 illustrates the laying of grids in this case) is possible, it is accompanied by technological difficulties due to the instability of the structure during assembly (compression) - grids tend to take a more stable position, spontaneously shifting in neighboring layers by half the size of the cell to the light (Figure 7). Such a structure of the FMTS (let us call it a dense regular structure or a structure with a dense packing of grids) was realized by us in our studies.

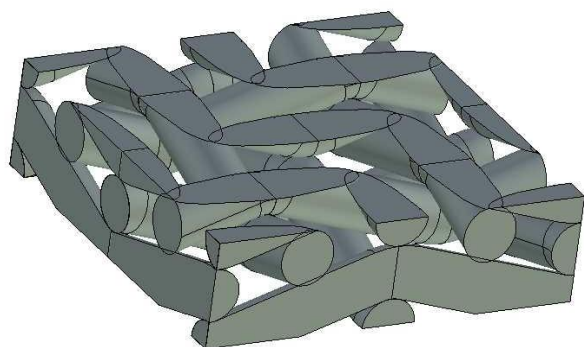


Fig. 6. 3D-model of the simple regular FMTS structure

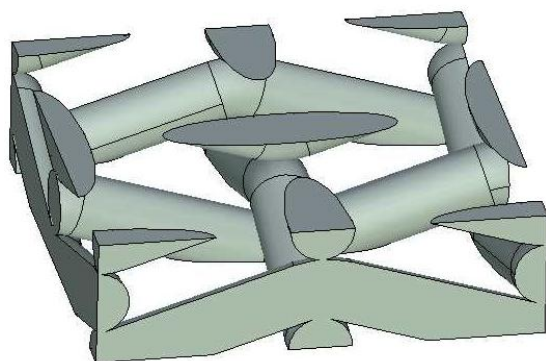


Fig. 7. 3D-model of the dense regular structure of FMTS

It should be noted that it is advisable to use the initial meshes for the manufacture of products from FMCS with such a structure in the form of squares or rectangles - in this case, there is no need to manufacture special stamps or use expensive technologies for cutting blanks. Accordingly, the problem arises of developing a FMTS model with a dense regular structure and equations based on it for calculating the structural and hydrodynamic properties of a material.

At the same time, the products design from FMTS of a cylindrical shape with a centering longitudinal hole and an element tightening the flanges located in it (in the form of a stud, a perforated pipe with a thread, etc.) also has its advantages. It seems that a chaotic arrangement would be appropriate in this case neighboring grids with respect to each other (chaotic structure of FMTS or FMTS with chaotic grid laying), which also requires the development of an appropriate model and equations.

3 Conclusion

A comparative characteristics of experimental samples of filtering materials with an orthotropic structure based on woven meshes and standard paper filters used for fine purification of diesel fuel in automotive and tractor equipment analysis has been carried out. It's shown that FMTS with the same fineness of cleaning are able to provide the required fuel consumption. Promising directions for further research in the development of FMTS are given.

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Monte Carlo Simulation in Risk Assessment in Mathematical Generation of Long Data

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Abstract. The report will address the main problem in risk measurement, namely the lack of a sufficiently long series of data for the various variables, so that the trend of their development can be formed with negligible error. A model will be made through a mathematical calculation in order to derive a long enough series with statistics reflecting the correlation between the individual variable indices and the standard deviation (volatility) to be able to obtain results with greater accuracy.

Keywords: risk measurement, production and costs, Monte Carlo

Introduction

Monte Carlo simulation was created in the late 1940s by Stanislaw Ulam in partnership with his colleague John von Neumann. The Monte Carlo method allows all possible outcomes of decisions to be seen and the impact of risk to be assessed. This allows for more motivated decision-making under conditions of uncertainty.

The main idea of the Monte Carlo simulation in calculating the volume of risk (VaR - Value at Risk) is to construct a detailed picture of the portfolio risk by computer simulation of a huge number of random numbers possessing the main characteristics of the empirical portfolio return distribution for a certain period [1,2]. One of the main statistical characteristics of asset returns is their random distribution. The change in their future prices is unknown, and usually the best guess for the next period's price is the last known price [3,4,5]. When such a variety of random changes in asset prices is observed, one logical solution is to determine the size of this change (market risk) by simulating a large number of random variables with certain characteristics. In this way, a complete picture of the price dispersion is constructed with an emphasis on the last known price. The use of random variables accompanied by the corresponding probabilities allows to calculate all possible variations in the price of the asset. This is precisely the main idea behind the Monte Carlo method for calculating VaR. The main stages through which the calculation of VaR passes when using Monte Carlo simulation are the following:

- Acquisition of up-to-date historical data for the assets with a sampling horizon of 1-3 years;

- Investigating the shape of the empirical distribution and constructing a mathematical model;
- Calculation of changes in production indices, arithmetic mean, standard deviation and correlation-variation matrix;
- Computer simulation of a minimum of 1,000 to 10,000 random numbers with defined statistical characteristics;
- Converting the random variables into sampling, by using certain statistical distributions;
- Simulating (calculating) the resource change in the production portfolio;
- Simulating the dynamics in the value of the entire portfolio of production costs and finding its difference compared to the value planned at the initial stage;
- Determining the value of VaR as a certain percentage with a certain percentage of accuracy;
- Analysis of the results and selection of a scenario to influence the factors with the highest volatility.

Essentially, simulation is a computerized mathematical technique that allows people to account for risk in a quantitative analysis. It provides all possible outcomes for a given choice of actions and shows how likely they are to occur. Applied to a production portfolio of activities and the indices on which they depend, this means that it is possible to use Monte Carlo simulation to help analyze all the risk factors. It shows a range of results from the best possible index change scenario to the worst possible one. And of course, it can also show what would happen if decisions were made "in the middle of the

road". This is especially useful for managers who want to analyze possible options [6,7,8].

Monte Carlo simulation builds models of potential outcomes by substituting a range of values for each uncertain factor. This is known as a *probability distribution*. The simulation then cycles through all possible outcomes, using a different set of random values each time. This can take tens of thousands of calculations. During a Monte Carlo simulation, values are drawn randomly from the input probability distributions. Each set of samples is defined as a *replicate*. The result obtained from each sample is then recorded.

1. Research Methodology and Results

For the purpose of the study, four indices will be taken for possible changes in production costs, as we have data on their movement for a period of two years. The data is displayed for weekly periods that correspond to the weekly periods of the planned cost matrix. Example data can be seen in Table 1:

Table 1. Historical changes in manufacturing indices by week

HISTORICAL VALUES			
Index1	Index2	Index3	Index4
100.00	100.00	100.00	100.00
101.31	101.10	99.88	99.30
99.04	100.54	100.72	99.12
100.73	101.03	99.95	97.74
97.57	100.32	100.57	101.81

The initial plan value for each index is taken as 100%, and the rest are formed according to the formula:

$Index\ 1 = 2\ period\ expenditure / 1\ period\ expenditure * 100$ (1)
etc.

In order to calculate the standard deviation of the assumed historical statistics, it is necessary to calculate an index of relative change, which is calculated as follows:

$LN\ (Index_{12}; index_{11}) = (index\ 12 - index\ 11) / index_{11}$
(2)
etc.

This results in an array of data for the weeks presented in Table 2.

Table 2. Historical changes in manufacturing indices by week

SIMULATED HISTORICAL CHANGES			
Index1	Index2	Index3	Index4
0.0025	0.0073	0.0029	-0.0134
0.0132	0.0026	0.0015	0.0178
0.0129	-0.0119	-0.0083	0.0028
-0.0584	-0.0038	0.0006	-0.0321

Examining the obtained data on historical changes between indices 1 to 4, two essential statistical indicators for the future model can be derived:

Coefficients of Correlation between Indices (Correlation Matrix) ;

Volatility (standard deviation of each index) .

1.1. Correlation and correlation matrix

A correlation is a statistical relationship between two random variables, whether it is intentionally caused or not. In the broadest sense, correlation is any statistical association, although it usually refers to the degree to which a pair of variables are linearly related.

In industry, correlation is an indicator that measures the dependence in the price movement of two resources or assets. A correlation coefficient is a statistical measure of the strength of the relationship between the relative movements of two variables. Its values range between -1.0 and 1.0:

- A correlation coefficient of -1 means a perfect negative correlation, i.e. asset prices move perfectly in opposite directions.

- A correlation coefficient of 1 means a perfect positive correlation, i.e. asset prices move perfectly in the same direction.

- A correlation coefficient of 0 means there is no relationship between asset price movements.

When it comes to types of correlation between two indices, they are basically three:

- Straight Correlation (Positive Correlation);

- Inverse correlation (negative correlation);

- Neutral correlation .

Direct correlation

Two assets have a direct correlation (positive correlation) when their values move in the same direction. Therefore, to be able to speak of a direct correlation, it must have a positive correlation coefficient (greater than 0).

Inverse correlation

An inverse correlation is observed when the parameter values move in opposite directions. So, to have an inverse correlation, you must have a negative correlation coefficient (less than 0).

Neutral correlation

When the prices of two assets on the financial markets have no relation to each other, the two assets can be said to be neutrally correlated. This happens when they have a correlation coefficient around 0.

In order to determine the correlation coefficients between the indices of relative changes, a correlation matrix was calculated in Excel .

Table 3. Correlation coefficients between indices of relative changes

CORRELATION MATRIX				
	Index1	Index2	Index3	Index4
Index1	1.0000	0.5761	0.0060	0.0711
Index2	0.5761	1.0000	0.5608	0.0933
Index3	0.0060	0.5608	1.0000	0.4368
Index4	0.0711	0.0934	0.4384	1.0000

Note: The diagonal is always one because it shows the index ratio to himself

1.2. Volatility – standard deviation

Standard deviation is a measure of the spread of a series or the distance from the standard. In 1893 Carl Pearson introduced the idea of standard deviation, which is arguably the most widely used measure, into scientific research.

It is the square root of the mean of the squares of the deviations from their mean. In other words, for a given set of data, the standard deviation is the root-mean-square deviation from the arithmetic mean. For the whole population it is denoted by the Greek letter "sigma (σ)" and for a sample it is represented by the Latin letter "s".

Standard deviation is a measure that quantifies the degree of dispersion of a set of observations. The further the data points are from the mean, the greater the variance within the data set, indicating that the data points are spread over a wider range of values, and vice versa.

Here is a quick overview of the steps involved:

Step 1: Find the mean of the array.

Step 2: For each value, find the square of the difference between the particular value in the data set and the mean value.

Step 3: Sum the values from step 2.

Step 4: Divide t by the number of values in the data set.

Step 5: Calculate the square root of the resulting amount.

Formula for calculating standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} \quad (3)$$

The syntax for calculating the standard deviation in Excel is as follows:

$$= STDEV (Index11:Index1n) \quad (4)$$

The sampling extent of the relative deviation data is illustrated in Table 4.

Table 4. Example of Volatility of production indexes by periods

Average	0.0001	-0.0001	0.0000	-0.0001	
Volatility	0.0222	0.0105	0.0054	0.0151	weekly
Volatility	0.0461	0.0217	0.0112	0.0312	monthly
Volatility	0.1596	0.0753	0.0388	0.1081	annually

Volatility, standard deviation results are calculated on a weekly basis. If for the purposes of the model they need to be transformed into data on a monthly or annual basis, the following transposition formulas can be used :

$$\text{per month} = (\text{Volatility Index n weekly}) * \text{SQRT}(1/7*30) \quad (5)$$

$$\text{for a year} = (\text{Volatility Index n monthly}) * \text{SQRT}(12) \quad (6)$$

Having the volatility for the selected time slice (for example e week) and the correlation between the factors is a base to use standart excel formulas to generate the adequate long line of variables that can run reliable Monte Carlo simulation. 1st step of the final part of the model is generates long line of Non-correlated normal distributed randoms using:

$$= \text{NORM.S.INV}(\text{RAND}()) \quad (7)$$

It is a pre-built integrated function in Excel that is categorized under statistical functions in Excel. The NORM.S.INV:

- The normal distribution is the most widely used in statistics. It is also called a "Bell curve" or "Gaussian curve."
- We can fully describe the normal distribution based on its mean and standard deviation (SD) values.
- A normal distribution is called "standard normal distribution" when its mean value is "0" or zero, and the standard deviation value is equal to 1.

The normal distribution can be standardized by using the below-mentioned formula:

$$z = (x - \text{mean}) / sd \quad (8)$$

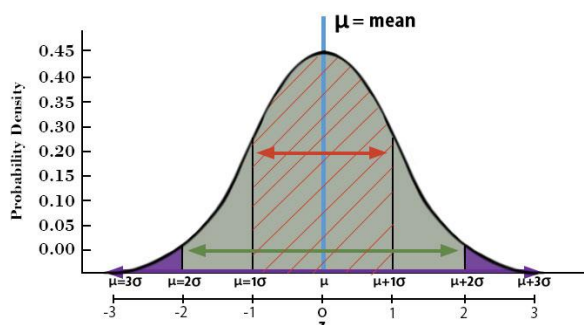


Fig 1. Bell Curve (Standart Deviation)

Z value on the x-axis is the standard normal deviation. Here, the red arrowed line in the curve indicates where the standard deviation of the mean value is within 1, whereas the green arrowed line in the curve indicates where the standard deviation of the mean value is within 2.

A set of normal distributed randoms (for example 5000) are generated for every risk index and time point by random generator, the result is a scenario matrix: (Indexes x time points) x scenarios, for example 4 indexes x 12 time points x 5000 scenarios. The matrix is then multiplied by Cholesky decomposition of the covariance matrix to receive correlated small changes of basic factors on time points. The simulation produces a large set of possible developments for the choosen indexes.

As a result, the model generates long enough Monte Carlo line of scenarios that contain volatility based correlated randoms for each time slice of the selected Indexes, pulled from the limited historical data.

Conclusions

In conclusion, it can be concluded that Monte Carlo simulation is particularly applicable to the manufacturing business, as it is often associated with random variables. It is used to assess the likelihood of cost overruns on large projects. Having enough historical data for each variable is a luxury for the management. That is why, analytics is using such mathematical models to create long enough line of data to run reliable Simulations that can help to determine the probability of the cost change and can help reduce the risk.

The use of Monte Carlo simulation can be useful as a window into the potential future of production costs. But it should not be taken as an absolute truth. It is a tool that would help in making asset allocation decisions, but it is important to

remember that markets can and likely will be volatile and unpredictable.

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Technological Aspects of grinding the side surfaces (Relieving) of the teeth Gear-cutting worm hobs

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Abstract. Contact line geometry at relieving of tooth flanks of gear cutting hobs and functional relations for calculation of relieving machine setting parameters for achieving the required accuracy of the tooth profile of the generating rack are determined. They include the setting angle of the grinding wheel axis and profiles of grinding wheel active surfaces for each of the tooth flanks. A method of calculating the single setting angle of the grinding wheel axis for both tooth flanks of the hob is developed.

Key words: radial relieving, tooth flanks of hobs, relieving machine, grinding wheel profiling, involute helical surface, curvature radii, curvature, contact lines

1 Introduction

The basis of the generating worm of a hob at cutting spur and helical gears of the involute meshing is the generating rack of the hob which is a set of cutting edges of teeth in one groove of the hob. When cutting the teeth of spur and helical gears, the generating worm of a cutting tool on a gear hobbing machine is formed by positioning the axis of the helical surface of tooth cutting edges in such a way that the generating rack in the machining area is parallel to the normal cross section (or perpendicular to the direction) of the gearwheel teeth to be cut. The helical surface of the hob on which the cutting edges of the hob teeth are located is the generating surface of the hob. The relieved surfaces of the hob flanks which form rear angles during cutting are analogs of helical surfaces with helix angles one of which is increased for one flank relative to the helix angle of the hob generating surface thread, and decreased for the other flank.

The nomenclature of hobs and deviations from the straightness of the cutting edges profile of the hob teeth are specified by the valid standards ISO 2490:20 07 Solid (monobloc) gear hobs with tenon drive or axial keyway, 0.5 to 40 module – Nominal dimensions and ISO 4468:2020 - Gear hobs - Accuracy requirements. These standards only apply to single-thread precision hobs.

In the technological process of hob production, relieving of flanks with a grinding wheel is still the most time-consuming operation. In the practice of tool production of hobs, a method of machining each of the flanks with the setting angle of the grinding wheel axis equal to the helix angle of the generating surface of the hob on the pitch cylinder has been established. At the same time, the geometry of generation of contact lines of the grinding wheel and the relieved surfaces has not been functionally determined until now. In order to obtain the required accuracy of tooth cutting edge profile, a long adjustment of the relieving machine by the setting and profiling parameters of the grinding wheel is carried out; trial grinding passes and intermediate control operations are performed. Thus,

the problems of relieving the tooth flanks of precision hobs for spur and helical gears remain relevant up to the present time [1-6].

The aim of the present work is, first of all, to clarify the geometry of formation of contact lines of grinding wheel surfaces and relieved surfaces during the operation of relieving the flanks. On the basis of this research for the operation of radial (the most widespread type) relieving we define functional relations for the calculation of the basic parameters of relieving machine setting to achieve the required accuracy of the tooth profile of the generating rack: the inclination angle of the grinding wheel axis relative to the horizontal plane and profiles of the grinding wheel active surfaces for machining each of the flanks. In particular, we define the method of calculating a single angle of the grinding wheel axis, so that the machine resetting for machining the second flank consisted practically only in replacing the grinding wheel (on the faceplate) with the appropriate active surface on the grinding spindle of the machine.

2 Basic parameters of hob flanks

Parameters of the ground surface are determined on the basis of the initial data: module m_0 of the generating rack (teeth of the cut gearwheel), pressure angle $\alpha_0 = 20^\circ$ of the generating rack profile, helix angle γ_{m0} on its pitch cylinder and axial angle α_{0x} of the profile of the generating surface of the hob:

$$\operatorname{tg} \alpha_{0x} = \operatorname{tg} \alpha_0 / \cos \gamma_{m0} \quad (1)$$

The value of the angle γ_{m0} is determined from the known relation:

$$\sin \gamma_{m0} = Z m_0 / d_{m0} \quad (2)$$

where d_{m0} is the pitch cylinder diameter. For a new hob you can accept $d_{m0} = d_{a0} - 2,5 m_0$ where d_{a0} is the standardized value of the hob external diameter.

To get the required profile axial angle α_{0x} of the generating surface of the hob with helical chip grooves (tooth faces), the relieved surface of teeth should have the profile angle α_x defined taking into account the lead P_z of the chip groove, the number of threads Z of the generating surface, the decline K of the relieved surface on one angular pitch of the hob teeth. For right side

hobs, the values of angles α_x for the right (R) and left (L) flanks are determined by the formulas [7-10]:

$$\begin{aligned}\alpha_{xR} &= \alpha_{0x} + 57,296 k \sin^2 \alpha_{0x} / [P + p + k \operatorname{tg} \alpha_{0x}]; \\ \alpha_{xL} &= \alpha_{0x} - 57,296 k \sin^2 \alpha_{0x} / [P + p - k \operatorname{tg} \alpha_{0x}]\end{aligned}\quad (3)$$

where: $P = P_z / 2\pi$ - is the helical parameter of the face surface, P_z is the lead of the chip groove (helical face surface);

$p = 0,5 \, m_0 \, Z / \cos \gamma_{m0}$ - is the helical parameter of the generating surface of the hob,

$k = K z_0 / 2\pi$ - is the parameter of relieving,

K - is the decline of the tooth back assigned in the direction of relieving movement (for radial relieving - perpendicular to the hob axis);

z_0 - is the number of chip grooves of the hob (or teeth in its cross section);

57,296 - is the coefficient of transition from the radian to degree value of the angular correction.

Thus, for a right-handed hob the axial angle α_{xR} of the right flank profile is greater than the axial angle α_{0x} of the generating surface profile of the hob, the angle α_{xL} of the left flank profile of teeth is less than the angle α_{0x} . For left-hand hobs, it is the reverse order:

$$\alpha_{xL} > \alpha_{0x} > \alpha_{xR}.$$

Helix angles $\gamma_{R,L}$ of both relieved flanks on the pitch cylinder of the right-hand hob are determined by the formula:

$$\begin{aligned} \text{tg } \gamma_R &= (p + k \text{tg } \alpha_{xR})/r_{m0}; \quad \text{tg } \gamma_L = (p - k \text{tg } \alpha_{xL})/r_{m0} \quad (4) \\ r_{m0} &= d_{m0}/2 - \text{is the radius of the hob pitch cylinder.} \end{aligned}$$

On an arbitrary radius r_i of lateral surfaces, the helix angles γ_i in each point of the profile are determined from expressions:

$$\begin{aligned} \operatorname{tg} \gamma_{\text{Ri}} &= (p + k \operatorname{tg} \alpha_{\text{xR}})/r_i; \\ \operatorname{tg} \gamma_{\text{Li}} &= (p - k \operatorname{tg} \alpha_{\text{xL}})/r_i \end{aligned} \quad (5)$$

For left-handed hobs, the indices $_{R,L}$ in formulas (3)...(5) are reversed.

3 Contact lines when grinding helical and relieved surfaces

There is the possibility of grinding of helical surfaces of worm threads and relieving of hob flanks by the grinding wheel as initially involute ones at parallel axes O_1-O_1 of the workpiece and O_w-O_w of the grinding wheel with the rectilinear axial profile. The contact line of the wheel and the ground surface touches the base cylinder and intersects the axis of the grinding wheel.

When grinding helical and relieved surfaces with a rectilinear axial profile by a disk grinding wheel, each of the points of the axial profile on the radius r_i of the ground surface with the assigned profile angle α_x and helix angle γ_i can be considered as an element of the involute helical surface. For each point of the axial profile, its base cylinder with radius r_{bi} can be defined. The angle α_{bi} of the ground surface profile in the plane tangent to the base cylinder is equal to helix angle γ_{bi} of the generatrix and profile angle α_{wi} of the grinding wheel in the same plane. The values of these angles are calculated by a known formula based on the axial angle α_x of the ground surface profile and the helix angle γ_i of the surface in the calculated point of the profile:

$$\operatorname{tg} \alpha_{\text{bi}} = \operatorname{tg} \gamma_{\text{bi}} = \operatorname{tg} \alpha_{\text{wi}} = (\operatorname{tg}^2 \alpha_x + \operatorname{tg}^2 \gamma_i)^{0,5} \quad (6)$$

We determine the radius r_{bi} of the base cylinder for each point of the axial profile as:

$$\begin{aligned} r_{bRi} &= (p + k \operatorname{tg} \alpha_{xR}) / \operatorname{tg} \alpha_{bRi}; \\ r_{bLi} &= (p - k \operatorname{tg} \alpha_{xL}) / \operatorname{tg} \alpha_{bLi} \end{aligned} \quad (7)$$

We set the axial angle α_x of the profile of the ground helical or relieved surface to be constant. At the section of the teeth adjacent to the hob tooth heads, the helix angle of the ground surface is the smallest for the profile due to the larger diameter, therefore, the angle $\alpha_{ba} = \gamma_{ba} = \alpha_{wa}$ of the profile and inclination of the generatrix of involute surface is the smallest, so here the radius r_{ba} of the base cylinder will be the largest for the ground profile. For the section of tooth roots the similar angle $\alpha_{bf} = \gamma_{bf} = \alpha_{wf}$ is the largest, respectively, the radius r_{bf} of the base cylinder will be the smallest.

Let us refer to Fig. 1. When the axes O_w-O_w of the wheel and O_1-O_1 are parallel, all the tangents to the base cylinders of the points of the tooth profile converge on the axis of the grinding wheel [9, 10].

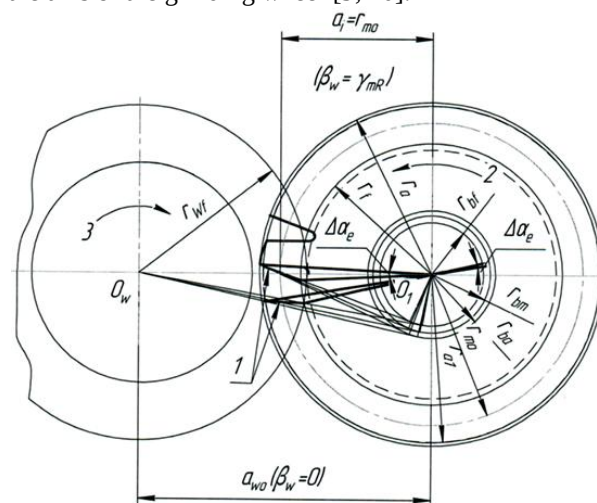


Fig. 1. Generation of contact lines when grinding the surface with a rectilinear profile

- 1 - contact lines of the straight section of the profile,
2 - direction of rotation of the cutter,
3 - direction of rotation of the grinding wheel

The assigned point of the axial profile of the ground surface on the contact line of the wheel surfaces and the workpiece is obtained at the intersection point of a tangent to the base cylinder and the cylinder, on which the assigned point is located. All tangents to base cylinders corresponding to the axial profile points of the ground surface converge in one point of the interaxial perpendicular, and the enveloping line of contact points obtained in this way is the locus of points of common normal lines to these surfaces and it determines the contact line of the surfaces of the wheel and the workpiece.

When the inclination angle β_w of the grinding wheel axis relative to the workpiece axis is changed, tangents to the base cylinders of the ground surface also change their position. Thus, at the value β_w equal to the helix angle of the ground surface on the pitch cylinder, the contact line of the wheel and the ground tooth surface crosses the interaxial perpendicular at the point on the pitch cylinder, and, thus, all the tangents to base cylinders of the ground surface profile points cross the interaxial perpendicular in the same point.

Obviously, the tangents to base cylinders of the ground surface and the contact line of the latter with the

grinding wheel surface may also occupy other positions relative to the interaxial perpendicular, depending on the value of the angle β_w of the grinding wheel axis setting. And the position of the point of the interaxial perpendicular, where all tangents to base cylinders corresponding to profile points converge, determines the value of the inclination angle β_w of the grinding wheel axis relative to the workpiece axis. If the inclination angle β_w of the wheel axis is equal to the helix angle γ_i of the ground surface within the height of the ground profile, then the intersection of the contact line of the wheel and the ground surface with the interaxial perpendicular takes place within the height of the ground profile at the point on the radius r_i where these angles are equal.

If the inclination angle of the grinding wheel axis differs from the values of angles of elevation of the ground surface, then the tangents to the base cylinders of points of the ground profile intersect with the interaxial perpendicular outside the profile of the ground surface, at a distance a_i from the hob axis. And for a given value of the angle β_w , it should be determined (for example, for the right flank of the teeth of a right-handed hob) from the expression:

$$a_i = (p + k \operatorname{tg} \alpha_{xR}) / \operatorname{tg} \beta_w \quad (8)$$

If the angle β_w is set equal to the pitch helix angle γ_{m0} of the generating surface of the hob, which is defined as $\operatorname{tg} \gamma_{m0} = p / r_{m0}$, then formula (8) will be as:

$$a_m = r_{m0} (1 + k \operatorname{tg} \alpha_{xR} / p) \quad (9)$$

The length and configuration of the contact line inside the extreme points of the ground profile with a change in the angle β_w are practically not changed, since the central angle $\Delta\alpha_e$ covering the contact line of the grinding wheel surface and the ground surface is unchanged. Its slope and distance are changed relative to the interaxial perpendicular (Fig. 1). For the left side of the teeth of the right-handed hob (on the right side of the root, if you look at the front surface of the teeth) in formulas (8), (9) we should replace the sign (+) with the sign (−) and the index (R) with (L) in the notation of the angle α_x .

3 Calculation of the grinding wheel profile along the contact line

Let us refer to Fig. 2, which shows the axial section of the grinding wheel. At parallel axes of the grinding wheel and hob, knowing from (6) angles $\alpha_{wi} = \alpha_{bi} = \gamma_{bi}$ of the grinding wheel profile in the key points of the grinding surface profile - on cylinders of heads α_{wa} , pitch α_{wm} and roots α_{wf} - you can define radii R_f , R_a of curvature of the axial profile of the grinding wheel.

In the plane tangent to the base cylinder, at the profile point of the hob on its pitch cylinder, the contact point of the wheel profile has the profile angle α_{wm} . This point of the wheel profile is the junction point of the curve sections of radii R_a and R_f . The length of working sections of the wheel profile is defined as $m_0 / \cos \alpha_{wm}$.

The radius R_a of the curvature of the ac surface of the grinding wheel machining the hob tooth head is defined as:

$$R_a = (m_0 / \cos \alpha_{wm}) / \sin (\alpha_{wm} - \alpha_{wa}) \quad (10)$$

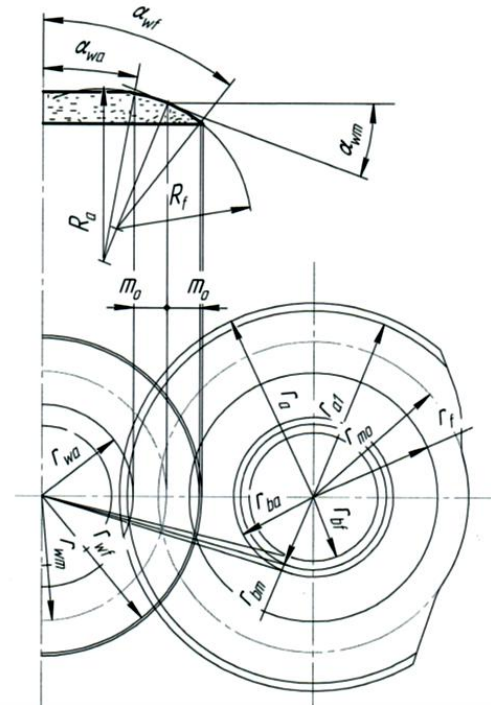


Fig. 2 Determining the grinding wheel profile along the contact line

Radius R_f of curvature of the active surface of the grinding wheel machining the hob tooth root is defined as:

$$R_f = (m_0 / \cos \alpha_{wm}) / \sin (\alpha_{wf} - \alpha_{wm}) \quad (11)$$

Convexity arrows δ_a and δ_f of the wheel profile in these sections are determined by the formulas:

$$\begin{aligned} \delta_a &= (m_0 / \cos \alpha_{wm})^2 / 2R_a = 0,5m_0 \sin (\alpha_{wm} - \alpha_{wa}) / \cos \alpha_{wm}; \\ \delta_f &= (m_0 / \cos \alpha_{wm})^2 / 2R_f = 0,5m_0 \sin (\alpha_{wf} - \alpha_{wm}) / \cos \alpha_{wm} \end{aligned} \quad (12)$$

Thus, at $\beta_w = 0$ (parallel to the axes of the wheel and hob) the contact line defined above and the specified arrows of convexity of the grinding wheel profile compensate the deviation of the axial profile of the involute relieved surface relative to the rectilinear axial profile of the Archimedean relieved surface, which, strictly speaking, is what we aim to obtain.

The radius R_f is somewhat smaller than the radius R_a of the curvature of the section machining the tooth head. Accordingly,

$$\delta_f > \delta_a \quad (13)$$

If the difference in values $(\delta_f - \delta_a)$ is within the tooth profile tolerance of ISO 4468:2020 (for a certain hob range), then the axial profile of the wheel can be dressed to a single radius of curvature, preferably to the R_a value. Since in this case the profile of the relieved tooth surface (and the cutting edge) on the hob tooth root closer to the cavity will be more complete (tooth profile deflection in +), and the gear head on the hob will have a small flange. This is in principle acceptable by the standard; and it helps to improve the characteristics of gear meshing in terms of the noise level.

The same profile of the grinding wheel can be used when setting the angle of the grinding wheel in the value β_{w1} :

$$\tan \beta_{w1} = (p + k \operatorname{tg} \alpha_{xR}) / a_{w0} \quad (14)$$

where a_{w0} is defined as follows (Fig. 3).

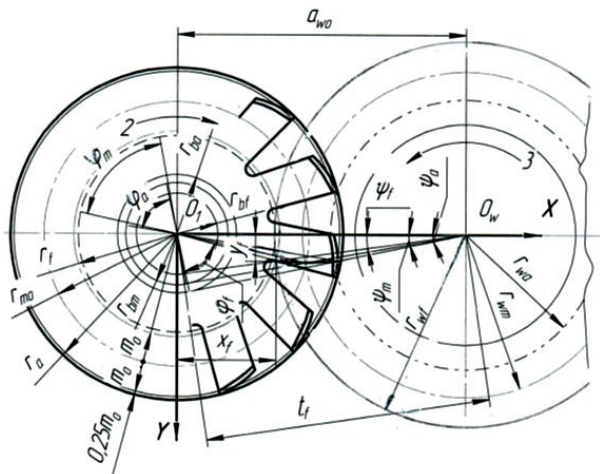


Fig. 3 Determination of coordinates of contact lines
1 - contact line, 2 - direction of rotation of the cutter,
3 - direction of rotation of the grinding wheel

It is most convenient to do it with the help of a tangent to the base cylinder of radius r_{bf} of the point of the grinding surface profile on the cylinder of roots of radius r_f , as at this point there is a contact with the grinding wheel on its maximum radius r_{wf} of the active surface. From the axis of the hob, we draw a perpendicular to the tangent of the base cylinder. The tangent passes through the root profile point at radius r_f . The length of the tangent t_f is defined as:

$$t_f = r_{wf} + r_{bf} \operatorname{tg} \varphi_f \quad (15)$$

where φ_f is the angle between the radii r_{bf} and r_f determined from the expression:

$$\cos \varphi_f = r_{bf} / r_f \quad (16)$$

Similarly, we find the values of the angles φ for the profile points on the head of the hob tooth φ_a and its pitch cylinder φ_m from the expressions:

$$\cos \varphi_a = r_{ba} / r_a; \quad \cos \varphi_m = r_{bm} / r_{m0} \quad (17)$$

The maximum radius r_{wf} of the grinding wheel is taken either from the technical data of the relieving machine: 60 mm for hobs with a diameter d_a up to 160 mm and module up to 12 mm, or 87 mm - for hobs of greater dimensions, or by measuring the current working diameter of the grinding wheel.

Then determine the angle ψ_f between the tangent and the interaxial perpendicular from the expression:

$$\operatorname{tg} \psi_f = r_{bf} / t_f \quad (18)$$

and we find the value a_{w0} by formula:

$$a_{w0} = r_{bf} / \sin \psi_f \quad (19)$$

4 Contact of the grinding wheel and tooth surfaces when setting the grinding wheel axis to assigned angle

With an arbitrarily assigned angle β_w of the grinding wheel axis, the characteristic of the grinding wheel active surface contact with the relieved surface of the hob teeth is changed. The length of the contact line and its configuration within itself are constant, because the values of the angles φ_i at contact points depend only on the parameters of the relieved surface - the radii of base cylinders and radii of the cylinders of the location of these points.

However, when the angle β_w is changed, the contact line rotates relative to the hob axis and shifts relative to

the grinding wheel axis. The distance a_i from the hob axis to the intersection point of the tangents to the base cylinders on the interaxial perpendicular does not coincide with the dimension a_{w0} .

As a consequence of the above, the angle μ_β is formed between the tangent to the base cylinder of the contact point on the pitch cylinder of the relieved surface and the axial section of the grinding wheel, passing through the same point of the axial profile of the tooth surface (Fig. 4). And to calculate the axial profile of the grinding wheel, this change in position of the contact line of the wheel and the relieved surface should be taken into account.

By formula (8) we determine the value of a_i , then for the key points of the profile (t_i, m, a) we find:

angles ψ_i between the tangents to the base cylinders and the interaxial perpendicular from the expression

$$\sin \psi_i = r_{bi} / a_i \quad (20)$$

lengths t_i of the tangents to the base cylinders

$$t_i = r_{bi} / \operatorname{tg} \psi_i \quad (21)$$

lengths of the segments b_i from the point of intersection of the tangents with the interaxial perpendicular to the corresponding point of the ground surface profile:

$$b_i = t_i - r_{bi} / \operatorname{tg} \varphi_i \quad (22)$$

and then you can calculate the coordinates X_i, Y_i of points of the contact line by the formulas:

$$X_i = a_i - b_i \cos \psi_i; \quad Y_i = b_i \sin \psi_i \quad (23)$$

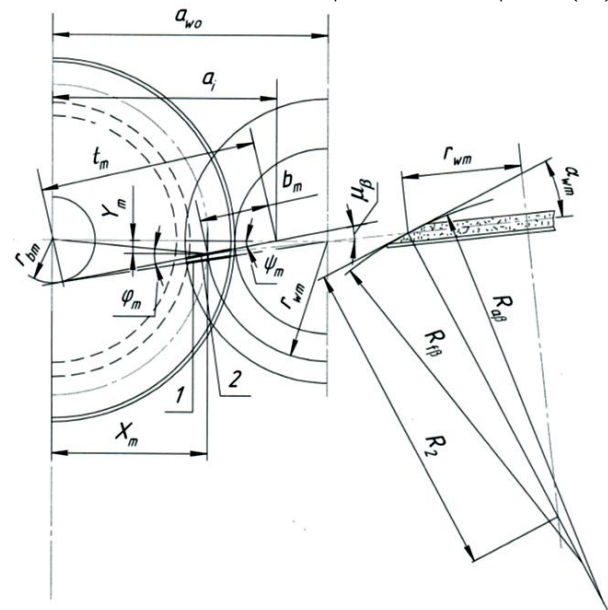


Fig. 4 Determination of the contact line and axial profile of the grinding wheel at $\beta_w \neq 0$

1 – contact line at $a_{w0}, \beta_w = 0$; 2 – contact line at $a_i, \beta_w \neq 0$

In general case we refer parameters $R_a, \delta_a, R_f, \delta_f$ of curvature of the grinding wheel profile defined by formulas (10) ... (12) to the normal section of the grinding wheel surface in the plane, tangent to the base cylinder of the profile point on the pitch cylinder, as the contact point on the pitch cylinder is a junction point of two sections of the grinding wheel profiling. The value of the angle μ_β of the deviation of the plane, in which the curvature parameters of the grinding wheel are defined, from the axial section of the grinding wheel will be determined from the relation:

$$\mu_\beta = \operatorname{acos} (Y_m / r_{wm}) - (90^\circ - \psi_m) \quad (24)$$

where $r_{wm} = (r_{wf} - m_0)$ is the radius of the grinding wheel at the contact point on the pitch cylinder of the hob.

To determine the parameters of the axial profile curvature of the grinding wheel at an arbitrarily assigned angle β_w we will use the Euler formula, which describes the ratio of curvatures of lines on the surface: for each normal section of the surface its curvature $1/R$ is defined as [11]:

$$1/R = (\cos^2 \mu / R_1) + (\sin^2 \mu / R_2) \quad (25)$$

where: R_1 and R_2 are the basic radii of curvature, i.e. the largest and the smallest values of R ; they are obtained at the basic normal sections;

μ is the angle between the planes of normal sections with the basic radius of curvature R_1 and the current radius of curvature R ; in this case, it is between the axial section of the grinding wheel and the section tangent to the base cylinder, where the radii of curvature are defined by formulas (10) and (11).

$$\operatorname{tg} \mu = \operatorname{tg} \mu_\beta \cos \alpha_{bm} / \cos \beta_w \quad (26)$$

With respect to the question under consideration, in formula (25) R are the radii R_a and R_f of the curvature sections in the defined (25) normal section to the surface of the grinding wheel along the contact line found by formulas (10) and (11); R_1 are the sought radii of curvature of the axial section of the wheel, corresponding to the profiling sections for the heads and roots of teeth; R_2 is the radius of curvature of the grinding wheel in the plane perpendicular to the axial profile of the grinding wheel at the contact point on the hob pitch cylinder. R_2 is determined by the formula:

$$R_2 = r_{wm} / \sin \alpha_{wm} \quad (27)$$

The value of the angle α_{wm} of the grinding wheel profile in its axial section at the contact point with the hob pitch cylinder is determined from the relation:

$$\operatorname{tg} \alpha_{wm} = \operatorname{tg} \alpha_{bm} \cos \mu \quad (28)$$

The values of sought radii $R_{a\beta}$ and $R_{f\beta}$ of curvature (as R_1) in the axial section of the grinding wheel are obtained from (25):

$$R_1 = R_{a,f} R_2 \cos^2 \mu / (R_2 - R_{a,f} \sin^2 \mu) \quad (29)$$

Our experience showed that when grinding the face of the hobs of the most widespread type Version 2, for the flank of these hobs where the helix angle of the generating surface is greater than the helix angle of the relieved surface (for example, the left flank of the hob teeth with the right direction of the generating surface), the accuracy of the profile of the generating rack is ensured with a multiple margin for all the hob types when grinding with a straight profile, and the contact line is also practically straight. For the opposite flanks (right flank of the tooth of a right-handed hob) in hobs with modules up to 2.25 mm, the accuracy of the flank profile is also technologically ensured according to accuracy class 4A without the use of radius dressing. With increasing the module, depending on the required accuracy class of the hob, radius dressing of the grinding wheel profile is required according to the above method.

5 Determination of a single angle of inclination of the grinding wheel axis

Setting of the grinding wheel axis inclination angle, which is uniform for grinding both flanks, provides

reducing the machine readjustment to the change of grinding wheel on the machine spindle and thus limiting the time of readjustment only by achieving the required profile of the grinding wheel active surface. In addition, when using machines with a dressing mechanism that allows profiling both sides of the grinding wheel, such a setting will make it possible to produce a double-sided relieving of teeth of worm hobs.

The conditions for a single angle of the grinding wheel setting for grinding both relieved surfaces are: firstly, the intersection of the grinding wheel axis by normal lines to these surfaces; secondly, a single starting point for grinding both flanks of the teeth.

The beginning of tooth machining on each flank takes place at the apex of the tooth, since it is the apex of the tooth that initially crosses the contact line of surfaces of the hob tooth and the grinding wheel. In addition, on the side where the helix angle is greater than that of the generating surface of the hob (on right flanks of right-handed hobs), due to greater values of the radii of the base cylinders of profile points, there is a more extended area of tooth entry to the full grinding height and, accordingly, the length of the relieving stroke and the arc of hob rotation during this stroke. In this regard, a single starting point for both flanks as a necessary condition should be defined by crossing the contact lines of the right and left flanks not on the outer diameter of the straight section of the tooth profile, but at the point near the pitch cylinder, where there is the middle of the infeed section on both flanks; and where the helix angles of the generating surface of the hob and the front surface of teeth coincide.

Our research has shown that the desired value of a single angle of installation of the axis of the grinding wheel is almost very close to the angle of elevation of the producing surface on the dividing cylinder with an acceptable deviation in the direction of its decrease by $0,1^\circ \dots 0,15^\circ$ to the value fixed by setting of the angle on the scale of the machine.

6 CONCLUSIONS

1. A method of constructing the contact lines between grinding wheel surfaces and ground flanks of hob teeth on the basis of considering each of the points of axial profile of the relieved surface as an involute helical surface element has been found.
2. It is determined that all the tangents to the base cylinders corresponding to the axial profile points of the ground surface, converge at one point of the interaxial perpendicular, and the enveloping line of the contact points thus obtained is the locus of the points of common normal lines to these surfaces and determines the contact line of the grinding wheel surfaces and the hob teeth.
3. The length and configuration of the contact line inside the extreme points of the ground profile with changing the inclination angle β_w of the grinding wheel axis is practically not changed, as the central angle covering the contact line of the grinding wheel surface and the ground surface is unchanged. Its inclination and distance relative to the interaxial perpendicular is changed.
4. A procedure for calculating the grinding wheel

profile for obtaining a rectilinear profile of the relieved surface is developed, including the case when the contact line does not coincide with the axial section of the grinding wheel.

5. It is determined that the angle of inclination of the axis of the grinding wheel for processing both sides of the teeth is $0.1^\circ \dots 0.15^\circ$ less than the angle of elevation of the producing surface of the milling cutter on the dividing cylinder

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Theoretical Research of Transmissions and Gearboxes

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Abstract. Significant aspects of the consequences of global warming and some measures that can be undertaken in order to reduce energy consumption and harmful emissions have been presented. The most important actions in this field are related to the renewal of the design of automobile vehicles and their components. A theoretical survey has been carried out in two main areas: investigating possibilities of improving the operational characteristics and the design parameters of transmissions and gearboxes, aiming to identify the areas of future dissertation research. A systematization of the reviewed scientific publications has been carried out. Main areas in which additional research can be carried out, related to the specific features of the Bulgarian industry and the state of the available cars in usage, have been identified.

1 Introduction

In order to reduce the consequences of global warming and to strengthen the empathy of countries to climate change, the Paris Agreement [1], [2] formulated important actions and measures aiming to reduce harmful emissions and to control the increase of environmental temperature.

As it is known, the largest concentration of carbon monoxide, nitrogen oxides, ozone and other chemical compounds is observed in the central part of the cities. In [3] and [4], the authors create an extremely significant mathematical models of four analysed case studies, through which they offer recommendations for reducing pollution in an urban environment.

The authors teams of [5], [6], [7] and [8] investigate and justify several particularly important and applicable measures that can be undertaken to reduce energy consumption and emissions, the most important of which is related to the design renewal of automobile vehicles and their components.

Based upon the professional and academic experience of the author's team, it was decided to carry out a theoretical research in two main areas: investigating the possibilities of improving the operational characteristics and design parameters of transmissions and gearboxes.

The objective of the paper presented is to identify areas for future dissertation research concerning these extremely significant components of the automotive engineering design.

Besides, it is necessary to take into account the challenges related to the age and the technical characteristics of the cars owned by Bulgarian citizens and the unfavorable consequences of the circumstance that the automobile industry in our country is not particularly very well developed.

2 Investigating the possibilities for improving the operational characteristics of transmissions and gearboxes

The possibilities for theoretical and experimental research of transmissions and gearboxes are presented in a considerable number of publications. The most widely used experimental methods, described in [9] and [10], are based on the experimental determination of vibrations.

The modal analysis presented by the authors is a method related to vibration theory. Its main goal is to obtain the inherent characteristics of the structural system, such as natural frequencies and natural modes.

Due to the increasing requirements for comfort in the vehicle and the optimization of noise and vibration in the powertrain imposed by these conditions, the noise from the gearbox no longer has only a secondary role and it becomes a significant component of the car's interior noise. This circumstance is of particular importance for tonal noise frequencies in the teeth meshing, as indicated in [11] and in [12].

Very often the load mode analysis is performed using ANSYS, as presented in [13] and [14]. Research implemented by other scientists [15, 16], applies modal analysis of gearbox housings, taking into account some specific aspects of this approach.

The authors of [17] mainly focus on gear modal frequencies. The vibration modes of compound planetary gears as transmission components have been investigated by the authors' teams of [18] and [19].

3 Investigating the possibilities for improving the design parameters of transmissions and gearboxes

In order to define and select appropriate methods for determining the optimal design and kinematic parameters of transmissions and to study methods of monitoring and possibly system control towards better efficiency, it is necessary to apply appropriate analytical and computational tools to the problem.

In order to reduce fuel consumption and to increase autonomy, various types of engines and power transmission components have been investigated. Modern car manufacturers are focusing on new or optimized drive systems that can achieve minimum fuel consumption. These types of car design layouts are being developed not only for cars with internal combustion engines, but also for hybrid/electric vehicles [20].

The authors of [21] analyze the possibilities of a multi-stage gearbox to increase the torque, to reduce the acceleration time and to increase the achievable road gradient. A multi-speed transmission adapted to an electric drive has advantages in terms of energy consumption compared to its single-speed equivalent [21].

The approach suggested is characterized by the following main advantage: the probability of the drive system to operate in a higher efficiency range is greater according to [22, 23].

The development of other parameters of the drive design is reflected in the publications of university scientists and representatives of the automotive industry. An interesting analysis is proposed in [24]. The relationship between clutch load, speed and torque during the shifting process is examined. Two adaptive control strategies are proposed, which according to the authors, can effectively compensate clutch coupling in a suitable time range.

The implementation of design improvements of transmissions and gearboxes is carried out with suitable modern CAD systems (Catia, ProEngineer, Unigraphics) in established manufacturing companies in the field of vehicle drive systems and combustion engines, [25].

Based upon CAE company products and authors' software from the field of MBS and FEM, extremely precise calculations can be applied. The objective of research in these areas is to minimize vibrations and to optimize the geometry parameters of the gearbox housing concerning transmission-specific excitations, [26, 27].

An important additional result considering contemporary design is the precise determination of shaft deformation, which can indicate misalignment. The highest level of precise calculation is achieved through the integration of flexible components in the MBS (multi-body simulation) model.

The vibration behaviour of the elements is described in details in [28, 29] and it becomes possible to perform resonance and strength calculations, local stress values and safety boundaries.

4 Conclusions and future work

The theoretical analysis made can be summarized in the following way. The authors' team considers that the future research of the PhD student will continue the scientific work described in details in previous works of scientists at the University of Ruse. These publications are mainly based on current STEM research achievements, [30-33].

Besides, the circumstance is to be taken into account that the automotive industry in Bulgaria is not particularly well developed and due to this reason the equipment facilities for experimental research is limited in university and scientific research laboratories.

Due to these circumstances and thanks to the extremely active international activity of the University of Ruse, it will most probably be necessary to carry out part of the future experimental research of the PhD student in Romanian universities and laboratories.

The quality and opportunities of experimental equipment aimed at the problems of transport and the automotive industry have been described in a considerable number of publications of scientist from Craiova, Brasov and Pitesti, [34-43].

More than 40 modern scientific publications have been reviewed and they are systematized in Table. 1.

Table 1. Type and quantity of publications analysed.

Type	Quantity
Publication in the field of machine design	18
Publication in the field of automobile operational characteristics	25
Publication from the period from 2017 - 2023	16
Publication from the period before 2017	27

The thematic areas considered are current and topical. An extremely large number of scientists contribute with their research to the development of these areas.

It is determined that the following thematic areas are to be investigated within further research work of the PhD student: precise design applying CAD systems of elements of transmissions and gearboxes in order to improve their dynamic behavior.

Considering the technical state of the cars owned by Bulgarian citizens, approaches and criteria for improving the operational parameters of vehicles by methods that have not been analyzed in the world scientific literature are to be created.

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Spatial Motions Transformers for Incorporation into Robot Systems – Review of the Bulgarian Experience

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Abstract: This work is a review of research carried out by the author on the synthesis and design of the spatial gear mechanisms, oriented for incorporation in various technical devices, especially in robotic systems. It contains also a description of the innovative and inventive activities of the author over the years. The study illustrates the elaborated through the years, basic algorithms for the synthesis of spatial motions transformers (in particular Spiroid and Helicon gear sets), which are based on the mathematical model for synthesis upon a pitch contact point. The content of the solution of the two main tasks of the synthesis - synthesis of the pitch configurations and synthesis of the active tooth surfaces configures the mentioned above algorithm. Through it, real models of Spiroid and Helicon gear transmissions are synthesized and elaborated by 3D printing subsequently, which are designed for incorporation into robots of various purposes

Keywords: SPATIAL MOTIONS TRANSFORMERS, PITCH CONFIGURATIONS, 3D PRINTING, SMALL MODULE GEAR DRIVES

1. Introduction

The process of joining Bulgaria to the European Union is accompanied by continuous growth and the extreme importance of the increasing competitiveness of Bulgarian companies and their ability to withstand competitive pressure and market forces. That is why the implementation of scientific achievements and new technologies and the development of innovative potential are crucial for strengthening Bulgarian manufacturing and from there - increasing employment and achieving economic growth [1, 2]. Following the defined objectives from the program for economic reform, adopted by the European Council at Lisbon in 2000, extended in Gothenburg, and improved at Stockholm and Barcelona, the actions of the member countries of the European Union should be directed towards identifying the priority areas, among which the encouragement of innovations is an essential one. The Lisbon process requires to be found tools for the encouragement of competitive industries with the potential for future development that could have a significant impact on the change of the economy. A key tool to achieving the high competitiveness of a national economy is the development and consistent application of the policies for the

implementation of national innovations. Without going into details on the Bulgarian national innovation policy, we will note that by Decision №723 / 2004 of the Council of Ministers an Innovation strategy of the Republic of Bulgaria and the measures for its realization are accepted. The main goals of this first innovation strategy are related to creating conditions for stimulation of the scientific research with a view to creating innovative technologies and products and their subsequent implementation. In the discussed strategy, a series of measures for its implementation are intended. Among them, the most significant is to optimize the relationships: science-technology-innovation. By Decision № 875/2015 of the Council of Ministers the new project - an innovation strategy for smart specialization 2014 -2020 is approved. Its goal is: By 2020 Bulgaria should move into the group of "moderate innovators".

The objectives and measurements, contained in both national innovation strategies, are presented to a maximum extent by the author of this study in the field of hyperboloid gear transmissions. This current research provides a review of the author's approach through the last 20 years to the synthesis of crossed axes gear mechanisms, as well as their implementation as drives of actuators and transport devices into robotic systems.

2. Methods

2.1. Spiroid and Helicon gears- specific characteristics

The spatial transformation of rotations with constant angular velocities between fixed crossed axes is realized by three-link gear mechanisms, known as hyperboloid gears. This name is related to the form of their axodes (see Fig. 1), which are two mating hyperboloids of revolution, whose geometric axes coincide with the rotation axes. The contact line of the hyperboloids is the axis of the screw motion of the rotation links. The dimensions and the geometry of the hyperboloids, which are determined uniquely, depend on the parameters that are defining the dimensions and the structure of the gear set, as well as the gear ratio that characterizes the type of rotation transformation.

Skew-axes gear pairs transforming rotations using conjugate tooth surfaces are widely used among known motion transformers. They could realize the exact motion law (i.e. the gear ratio) they are designed to. The successful use of hyperboloid gears with new kinematic and strength characteristics in the industry makes slow progress because:

- common principles of transformation of rotations between skewed axes are not well examined;
- approaches to the mathematical modelling of synthesis of these types of mechanisms are not defined.

Spiroid¹ and Helicon gear pairs, which are the objects of this study, belong to one of the most progressive and comparatively less applied in the engineering practice type of skew-axes gears. These types of transmissions are known from [3-9, 11-13], etc. The first patents are introduced in the USA by Illinois Tool Works, Chicago. The firm has constructed the series of power classes and exactness tolerances classes of Spiroid gears and has organized the manufacture of Spiroid gear trains with a general function.

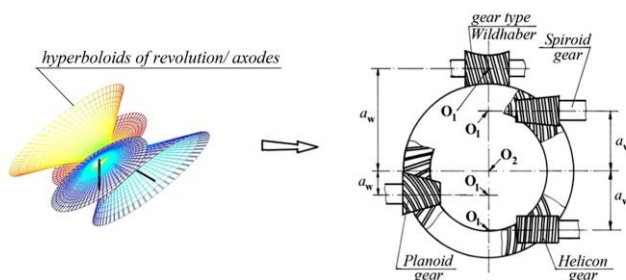


Fig. 1 Family of skew-axes gears

Among progressive combinations of spatial gears, Spiroid and Helicon gears occupy one essential place [14]. The placement of the mesh region, geometry, and technology of elaboration of these hyperboloid gears is the reason they are treated as a hybrid between hypoid gear sets and worm ones. For this reason, Spiroid and Helicon gear sets obtain constructive, technological, and exploitation characteristics that make them suitable for application as power and kinematic gear mechanisms, which ensure the smooth and noiseless rotations transformation and with a possibility to

realize mating without a backlash between mating tooth surfaces. A possible interval or reduction angular velocity ratios is $i_{12} \in [10, 200]$. This does not limit the possibility of obtaining gear sets with angular velocity ratios not belonging to this interval. The typical characteristic of the Spiroid and Helicon gears is a big number of simultaneously contacting teeth. The number of simultaneously contacting teeth of Spiroid and Helicon gears is 3-4 times bigger than for worm gears for angular velocity ratios in the interval $[10, 60]$. The mentioned specific characteristics of these gear sets are the main reasons for their successful integration into the construction of the robotic systems.

2.2. Synthesis of Spiroid and Helicon gears using a pitch contact point

Two approaches [1, 2], oriented to the mathematical modelling, to the synthesis of hyperboloid gear sets: „upon a pitch contact point” and „upon a mesh region” respectively, are known. Here, the focus will be put on the first approach, which is characterized by a certain universality and the possibility of realizing an optimization of the synthesized transmissions. The mathematical model for synthesis upon a pitch contact point is based on the assumption that the necessary quality characteristics (that define concrete exploitation and technological requirements to the active tooth surfaces) are guaranteed only at one concrete point P of the active tooth surfaces Σ_1 and Σ_2 , and in its close vicinity. According to this model, the common contact point P of the conjugate tooth surfaces Σ_1 and Σ_2 is the common point of both circles H_i^c ($i=1, 2$) which are called a pair of pitch circles $(H_1^c : H_2^c)$. The point P is called a pitch contact point, the plane T_m including the tangents to H_i^c ($i=1, 2$) at the point P – a pitch plane, and $m-m$ is the pitch normal to T_m at the point P . The pair of rotation surfaces that include the corresponding pairs of pitch circles and whose common normal at the point P is the straight-line $m-m$, are an analog of H_i^c ($i=1, 2$) and are called pitch surfaces H_i^s ($i=1, 2$). The pair of circles $(H_1^c : H_2^c)$ is directly related to the evaluation of the pitch and the tooth module of the designed gear set. The pitch circles and surfaces’ parameters define the dimensions of the reference surfaces, i. e. the blank proportions of the gears depend on them. These parameters are used when the mountings’ dimensions of the synthesized gear-set are calculated.

Thus, the mathematical model for synthesis based on a pitch contact point ensures the solution to two basic problems:

- synthesis of the pitch configurations;
- synthesis of the active tooth surfaces.

¹ Spiroid and Helicon are a trademark registered by the Illinois Tool Works, Chicago, Ill.

Pitch configurations synthesis: essence, definition, and mathematical modeling. The present paper treats the kinematic theory of the spatial rotations transformation in the context of defining the geometric and kinematic essence of basic elements of the mathematical models for the synthesis of hyperboloid gears and their role in solving the gears' synthesis. One of the most important elements are so-called pitch configurations (pitch circles and pitch surfaces) and deal with one actual but still disputable (as content and terminology) part of the meshing theory.

From the performed analysis of the above publications and the corresponding comments, it can be concluded that:

- If the law of transformation of rotations $i_{12} = \omega_1/\omega_2 = \text{const.}$ ($\bar{\omega}_i$ - angular velocity of the rotating link i) between fixed and crossed axes $1-1$ and $2-2$ (the shortest distance between them is $a_w = \text{const.}$ and the angle between them is $\delta = \angle(\bar{\omega}_1, \bar{\omega}_2) = \text{const.}$) is given, and if the position of a point (treated as a point of contact of conjugate tooth surfaces Σ_1 and Σ_2) in the fixed space is known, then the diameters and the mutual position of the circles H_i^c ($i=1, 2$) are completely and uniquely determined. The circumferential velocity vectors $\bar{V}_i$ ($i=1, 2$) of the common point P , and the relative velocity vector $\bar{V}_{12}$ at the same point, (i. e. the plane T_m where the coplanar vectors $\bar{V}_i$ ($i=1, 2$) and $\bar{V}_{12}$ lie), as well as the normal $m-m$ to T_m at the point P are determined uniquely, too.

- It is sufficient to know the mutual position of the crossed axes of rotation $1-1$ and $2-2$, and the position of the point P (as a common point of the tooth surfaces Σ_1 and Σ_2) in the fixed space, for the circles H_i^c ($i=1, 2$) to be completely and uniquely determined (as diameters and mutual position). The plane T_m formed by the tangents to the circles H_i^c ($i=1, 2$) at the point P , as the normal $m-m$ to T_m at the point P is uniquely determined, too. Thus, the following set of equations is obtained:

$$\begin{aligned} r_1 \cos \theta_1 &= r_2 \cos \theta_2 \cos \delta + a_2 \sin \delta, \\ r_1 \sin \theta_1 &= a_w - r_2 \sin \theta_2, \\ a_1 &= r_2 \cos \theta_2 \sin \delta - a_2 \cos \delta, \\ \cos \delta_1 \sin \theta_1 &= \cos \delta_2 \sin \theta_2, \\ \sin \delta_1 &= \sin \delta_2 \cos \delta + \cos \delta_2 \cos \theta_2 \sin \delta, \\ \cos \delta_1 \cos \theta_1 &= \sin \delta_2 \sin \delta - \cos \delta_2 \cos \theta_2 \cos \delta. \end{aligned} \quad (1)$$

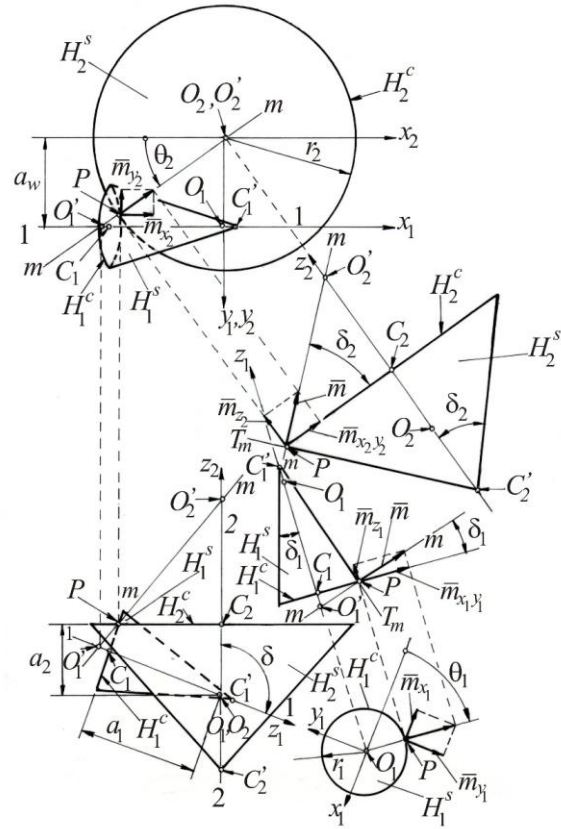


Fig. 2 Externally contacting pitch circles H_i^c ($i=1, 2$) and pitch surfaces H_i^s ($i=1, 2$) with a normal orientation, corresponding to hyperboloid gears with external meshing

Later, the study of the system (1) will be realized by taking into account the following geometric conditions

$$\begin{aligned} \theta_1 &\in \left[0, \frac{\pi}{2}\right], & \theta_2 &\in \left[0, \frac{\pi}{2}\right], & \delta_1 &\in \left[0, \frac{\pi}{2}\right], \\ \delta_2 &\in \left[0, \frac{\pi}{2}\right], & a_w &> 0, & r_i &> 0, & a_i &\geq 0, \quad (i=1, 2). \end{aligned}$$

Each of the last three equations in (1) is a consequence of the other two, i.e. (1) is a set of 5 independent equations with 10 unknowns: $\delta, a_w, \delta_1, r_1, a_1, \theta_1, \delta_2, r_2, a_2, \theta_2$. Therefore, each solution of (1) is a function of 5 of them (we will consider them as free ones). We suppose that the independent (free) parameters are $\delta, a_w, \delta_1, r_1$ and a_1 . We will look for analytical relations that have to be fulfilled so that set (1) has a solution. Let us pay attention not only to the cases oriented to the synthesis of Spiroid and Helicon classical interpretation gears [3 - 9] but also to such essential for the practice cases of orthogonal hyperboloid gears ($\delta = \pi/2$). For those two cases in the system (1) are substituted sequentially, when $a_1 \neq 0 - \delta_1 \neq 0$ (Spiroid drive), and $\delta_1 = 0$ (Helicon drive). Then after the calculation of the parameters, the corresponding geometric pitch configurations can be easily visualized [1, 13] as it is shown in Fig. 3 and Fig. 4.

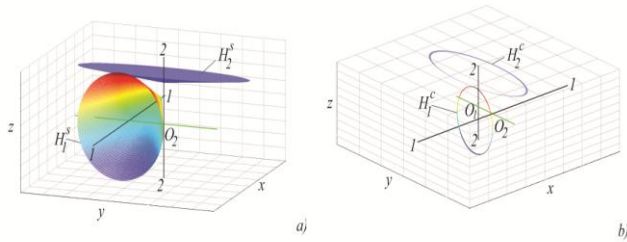


Fig. 3 Pitch configurations for orthogonal Spiroid gears: a) pitch surfaces H_i^s ($i=1, 2$); b) pitch circles H_i^c ($i=1, 2$); $a_w = 3,25$ mm; $\delta = 90^\circ$; $\delta_1 = 5^\circ$; $r_1 = 4$ mm; $a_1 = 9,5$ mm; $\delta_2 = 84^\circ$; $a_2 = 3,9983$ mm; $r_2 = 10,003$ mm

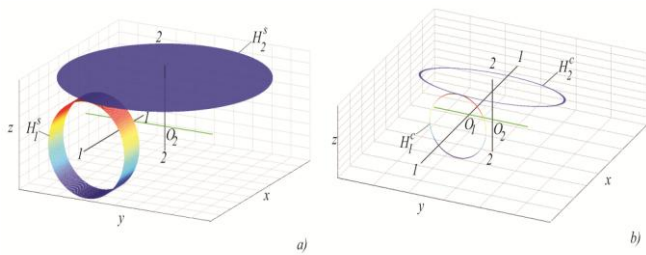


Fig. 4 Pitch configurations for gears of type Helicon: a) pitch surfaces H_i^s ($i=1, 2$); b) pitch circles H_i^c ($i=1, 2$); $a_w = 3,25$ mm; $\delta = 90^\circ$; $\delta_1 = 0^\circ$; $r_1 = 4$ mm; $a_1 = 9,5$ mm; $\delta_2 = 90^\circ$; $a_2 = 4$ mm; $r_2 = 10,041$ mm

Synthesis of Tooth Surfaces. The algorithm for the synthesis of pitch configurations is based on the principles of geometric synthesis. Therefore, it cannot directly define the kinematic characteristics of the synthesized mechanisms, since the law of motion transformation is not presented in the algorithm. When the law of motion transformation is included in the pitch configuration algorithm, the configurations start a common rotation. Thus, a sliding velocity vector $\bar{V}_{12}$ can be defined at their common point.

The magnitude and the direction of $\bar{V}_{12}$ are important since they determine the geometric parameters of the conjugated tooth surfaces Σ_1 and Σ_2 of the pinion and gear. The synthesis of the pinion thread surface Σ_1 is of essential interest when the manufacture of the gear-pair is based on Olivier's second principle. The synthesis of tooth surfaces upon a pitch contact point does not require knowing an analytic expression for the active flanks. The synthesis of the surface Σ_1 can be reduced to determine its basic geometric characteristics at the pitch point P . Thus, the synthesis algorithm comprises analytical relations defining a sufficient number of parameters of the active flanks Σ_1 by means of

which the conditions for the design of both the Spiroid pinion and the Spiroid hob are created. Here, should especially be noted, that the specific constructive characteristics, which are necessary for the design and elaboration of the Spiroid pinion and Spiroid hob with convolute, Archimedean, or involute active tooth surfaces of the pinion threads, are determined by using the calculated parameters in the pitch contact point. The Spiroid hob is used

for Spiroid gear Σ_2 tooth cutting. Here, some of the analytic relations included in algorithms and computer programs for Spiroid and Helicon gear synthesis are presented. Expressions given below correspond to gears with right-hand pinion threads and left-hand gear teeth. The directions of the angular velocities vectors $\bar{\omega}_1$ and $\bar{\omega}_2$, shown in Fig. 5, correspond to the mentioned orientation of the tooth surfaces Σ_1 and Σ_2 .

(a) Helix angle of the pinion thread flank. This angle gives the direction of the helix of the helical flank Σ_1 in the pitch point P . β_1 is the angle between the tangent $t-t$ to the helix L_1 in P and the straight line PO_1' passing through P and the concluding angle δ_1 with the axis $1-1$ (see Fig. 5):

$$\beta_1 = \arctan \frac{i_{12}r_1 - r_2 \cos \mu}{r_2 \sin \mu},$$

$$\mu = \arcsin \frac{\sin \theta_2 \sin \delta}{\cos \delta_1}.$$
(2)

(b) Axial helical parameter and axial module. The axial helical parameter is a basic geometric parameter of the pinion. By using it, the pinion helix pitch can be determined. Its expression is [1]:

$$p_s = \frac{r_1 r_2 \cos \delta_1 \sin \mu}{i_{12}r_1 - r_2 \cos \mu}.$$
(3)

Assuming $\delta = \pi/2$ and $\delta_1 = 0$ in (2) and (3), analytical expressions for β_1 and p_s corresponding to the Helicon pinion are obtained.

By using equation (3), it is easy to determine the axial pitch and axial module of the Spiroid pinion.

$$P_s = 2\pi \frac{p_s}{Z_1}, \quad m_s = 2 \frac{p_s}{Z_1}.$$
(4)

Here Z_1 is the number of Spiroid pinion threads (Z_2 - the number of Spiroid gear teeth).

The axial pitch and the axial module are basic geometrical characteristics of the gear sets of the considered class.

(c) Pinion thread pressure angles. When meshing mutually enveloping surfaces Σ_1 and Σ_2 , it is possible to obtain singular points in the mesh region. Such points exist if the relative velocities of the conjugate tooth surface contact points are zero [1]:

$$\bar{V}_{r,i} = \bar{0}. \quad (5)$$

Depending on the normal vector $\bar{n}_i$ to the meshed surfaces at their common point, two types of singular points exist:

singular points of first order called ordinary nodes if $\bar{n}_i \neq \bar{0}$, and consequently $\dot{\bar{n}}_{r,i} = \bar{0}$, where $\dot{\bar{n}}_{r,i}$ is the relative velocity vector of the top of the normal vector in its relative motion;

singular points of second order called points of undercutting when $\bar{n}_i$ does not exist.

Singular points of first order should be eliminated from the mesh region since they increase specific friction, reduce lubrication and heat transfer, and as a result, they decrease the loading capacity of the gear pair (because of the increased relative sliding).

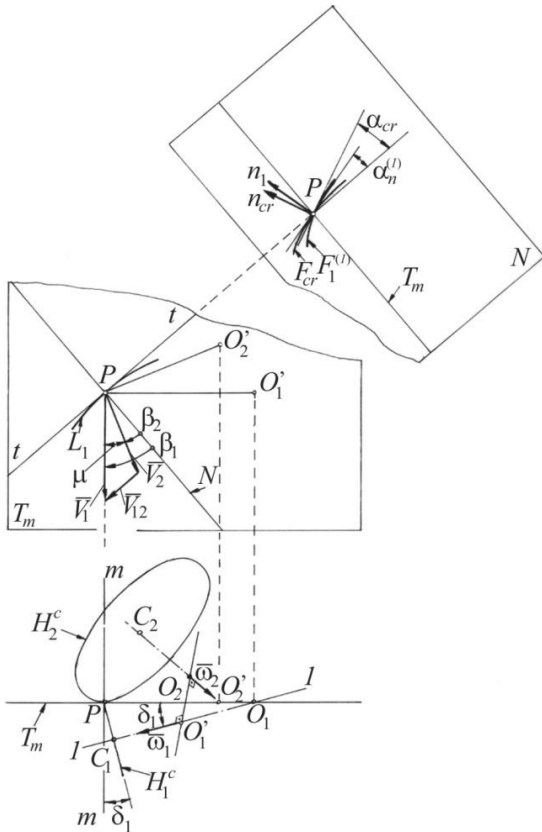


Fig. 5 Spiroid gear tooth geometry

In the cases of Spiroid gear synthesis, using a pitch contact point P , the aim is to eliminate the ordinary nodes in the pitch point vicinity.

To carry out this task, a limiting direction of the normal vector $\bar{n}_i$ with respect to the tooth surface Σ_1 at the point P is sought. Here, it is considered that a point of Σ_1 is an ordinary node. The analytic expression of the angle between this vector and the plane T_m (see Fig. 5) [1] is given:

$$\alpha_{cr} = \arctan \left\{ \frac{\sin \beta_1 [r_1 \cos \delta_1 + a_w \cot \delta (A) + a_1 (B)]}{r_1 \sin \delta_1 + a_1 \cos \beta_1 - a_w \cot \delta \tan \theta_1 \cos \delta_1} \right\},$$

$$A = \cot \beta_1 + \tan \theta_1 \sin \delta_1,$$

$$B = \cot \beta_1 \tan \theta_1 - \sin \delta_1. \quad (6)$$

The point P will not be an ordinary node if the normal vector $\bar{n}_i$ to Σ_1 shifts with respect to the critical normal vector $\bar{n}_{cr}$ by the tooth standard profile angle α . As a result, the pinion thread becomes non-symmetric in the normal section and the real profile angles in the normal section of the thread (i.e. the pressure angles) are of the form:

$$\alpha_n = \alpha \mp |\alpha_{cr}|, \quad (7)$$

The smaller value of α_n determines the orientation of the surfaces Σ_1 . They belong to the pinion threads when the pinion meshes with the Spiroid gear's corresponding surfaces Σ_2 and the angular velocity vectors are those shown in Fig. 5.

(d) Special force angle. The special force angle α_{12} is the angle between the vector of the force transferred from Σ_1 to Σ_2 and the circumferential velocity vector of the pitch contact point P , being a point of Σ_2 :

$$\cos \alpha_{12} = \cos \beta_2 \cos \alpha_n, \quad (8)$$

where $\beta_2 = \beta_1 - \mu$. The value of the special force angle has a critical effect on the conditions of force transmission between the conjugated surfaces Σ_1 and Σ_2 . It also affects the synthesized mechanical transmission efficiency. The basic geometric-kinematic characteristics, which are part of the analytical algorithm for optimization synthesis, are presented.

3. Conclusion

Technical applications and results. The described two main tasks, configuring the mathematical model for the synthesis of Spiroid and Helicon transmissions, as well as the based on them software program for the synthesis, are served to the authors for the realization of a series of driving for incorporations into different robotic systems: - Helicon gear pairs (three types), dedicated for the walking constructions of the walking robots with 6 and 4 insect type legs [10, 12, 14];

- Two types Helicon motor- reductor MRH 31 and MRH50 are incorporated into an experimental driving of a manipulator for proportioning and pouring of melts of aluminum and magnesium alloys in horizontal machines for casting under pressure FEEDMAT 1 [2, 14]; - Two types Spiroid and two types Helicon gear –pairs are synthesized and the elaborated models are realized by a 3D printing. They are intended for incorporation into a bio-robot hand [11]. Further below, only this technical result will be considered.

Spiroid and Helicon gears' models for driving into a finger of a robot-hand. Future humanoid robots, it is expected to perform different complex tasks by interacting with human users. These types of robots will be equipped with anthropomorphic multi-fingered hands that are similar to human hands. The main future designation of such a humanoid robot's hand is to replace the human presence when performing various dangerous tasks in the fields such as industrial manufacturing, chemical industry, space, seabed, etc.

Another perspective application of the anthropomorphic robot hand is its application in surgery as an extension of the doctor's hands and as a prosthesis for handicapped people. Such surgical hands that are designed for exact positioning and manipulation during spinal surgery will increase surgeon precision. Therefore, the requirement for such robot- devices (hands) is to obtain characteristics such as accuracy, smoothness, and exactness.

For this reason, a five-fingered robot hand is developed in Kawasaki&Mouri Laboratory at the Engineering Department of Gifu University [11]. The aim of the robot hand has to be used as the standard platform for the study of optimal grasping and manipulation of various types of objects. One of the tasks related to this hand is to find out a solution to the problems related to the increment of the overlapping coefficient and also to create preconditions for controlling the backlash between mating gears which are implemented into the fingers of this hand (see Fig. 6).

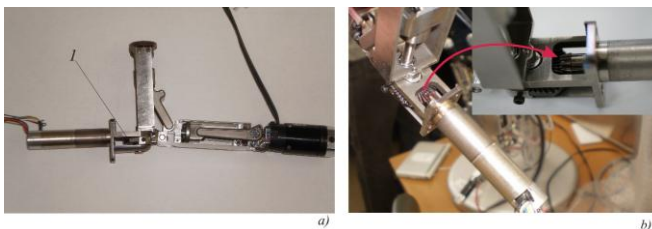


Fig. 6 Finger pattern of the robot hand: a) forefinger; 1-bevel gear; b) bevel gear with straight teeth: $m = 0,5$ mm; $i_{12} = 4$; $z_1 = 10$; $z_2 = 40$

The idea to incorporate Spiroid or Helicon gear drives into this robot hand is based on the following main points:

-These gears have a conjugate linear contact. They are characterized by a large number of simultaneously meshed tooth surfaces. This is a premise to expect smoother and noiseless rotations transformation, compared with the bevel gears with straight teeth. This is important for the precision

action of the gear set when a simple control is used on robotic systems.

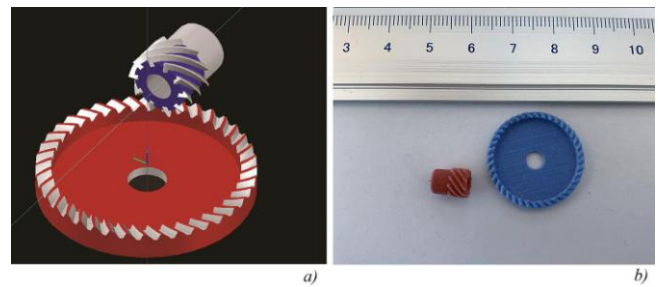


Fig. 7 Hyperboloid gear of type Helicon with: $i_{12} = 4$; $z_1 = 10$; $z_2 = 40$; $m = 0,5$ mm; $a_w = 4$ mm
a) 3D CAD model; b) 3D printed model (the shown scale is in mm)

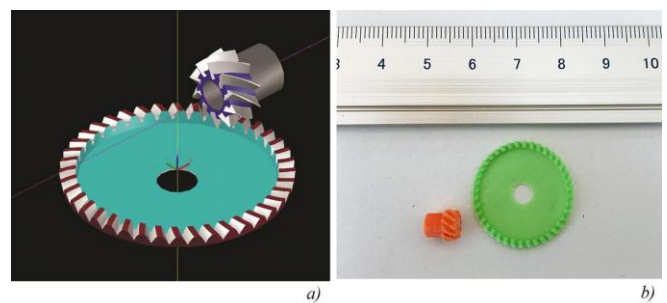


Fig. 8 Spiroid gear drive with $i_{12} = 4$; $z_1 = 10$; $z_2 = 40$; $m = 0,5$ mm; $a_w = 4$ mm; a) 3D CAD model; b) 3D printed model (the shown scale is in mm)

-The other essential characteristic of the considered class of spatial gears is the simplicity of backlash regulation. This is related to the constant axial pitch of the Spiroid pinion and the conic form of the Spiroid pinion reference surface. Thus the backlash control between the tooth flanks is obtained by pinion displacement along its axis. Since the Helicon gear is disk-shaped, the backlash control is performed by gear displacement along its axis. This is why these gear pairs are successfully applied when the accurate work of the mechanism requires meshing without backlash.

-The manufacturing technology of these types of gear drives ensures high accuracy of coincidence of the geometry of the pinion with the geometry of the hob that cuts the driven gear. This is a guarantee for the practical realization of gears with a minor deviation of the accomplished gear ratio from the given theoretical one. The purpose of the realized study is to replace a bevel gear (Fig. 6) with kinematically equivalent hyperboloid gears of type Helicon and type Spiroid. The gear set shown in Fig. 7 and 8 are specially synthesized by choosing the optimal structure and geometrical characteristics and it is CAD modeled and 3D printing manufactured. The goal is a smooth integration into the already existing robot hand, which will result in its technical precision. Activities related to its manufacture and integration into the robot are forthcoming. The authors of this study have experience in the development of gear drives of such type, as well as in the organization of their actual manufacture and the integration into industrial products [11].

4. Acknowledgment

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